

Homework 3 - Due October 3, 2012

1. Complete Problem 5 in Probability Online. You are given the file MontyHall.m. Update this file (see the question and comments in the file for more direction) and upload your code. You do not need to hand in anything on paper for this question.
2. Suppose that $n = a + b + c + d$. Show that the number of ways to place n objects into 4 bins with a in the first bin, b in the second, c in the third, and d in the fourth is

$$\frac{n!}{a! b! c! d!}.$$

Note: This fraction is denoted $\binom{n}{a, b, c, d}$ and is called a multinomial coefficient. In general, if $n_1 + n_2 + \dots + n_r = n$, then the number of ways of dividing n objects into r groups of size n_1, n_2, \dots, n_r is

$$\binom{n}{n_1, n_2, \dots, n_r} = \frac{n!}{n_1! n_2! \dots n_r!}.$$

Multinomials may show up in our class again.

3. (Section 4.1, Problem 1) Assume that E and F are two events with positive probabilities. Show that if $P(E|F) = P(E)$, then $P(F|E) = P(F)$.
4. (Section 4.1, Problem 6) From a deck of five cards numbered 2, 4, 6, 8, and 10, respectively, a card is drawn at random and replaced. This is done three times. What is the probability that the card numbered 2 was drawn exactly two times, given that the sum of the numbers on the three draws is 12?
5. (Section 4.1, Problem 17) Prove that if A and B are independent, so are
 - (a) A and \tilde{B} .
 - (b) \tilde{A} and \tilde{B} .

(continued on next page)

6. (Section 4.1, Problem 8) Let $\Omega = \{a, b, c, d, e, f\}$. Assume that $m(a) = m(b) = 1/8$ and $m(c) = m(d) = m(e) = m(f) = 3/16$. Let A, B , and C be the events $A = \{d, e, a\}$, $B = \{c, e, a\}$, $C = \{c, d, a\}$. Show that $P(A \cap B \cap C) = P(A)P(B)P(C)$ but no two of these events are independent.
7. In a certain city, 25% of residents are Independents, 35% are Democrats, and 40% are Republicans. In the mayor's race, 50% of Independents, 60% of Democrats, and 60% of Republicans voted.
- (a) Suppose you randomly select a person who voted in the mayor's race. What is the probability that he or she is an Independent?
- (b) What percentage of the city voted in the mayor's race?
8. (for 2 points) Answer Section 4.1, Problem 28. A few sentences countering the lawyer's argument is sufficient.

Practice problems NOT to turn in: 4.1.2, 4.1.7, 4.1.12, 4.1.19, 4.1.29, and the following problem:

Practice 1. Suppose your partner has two cards on the table.

- (a) Find the probability that both cards are aces, given that she has at least one ace.
- (b) Find the probability that both cards are aces, given that she has the ace of spades.