

Homework 4 - Due October 10, 2012

1. In a “Pick 4” lottery, a sequence of 4 numbers from 0 to 9 is chosen (e.g. 0860). It costs \$1 to play and pays out \$5000 if you pick the correct sequence of numbers. What is a player’s expected winnings?
2. (Section 6.1, Problem 16(a)) Gerolamo Cardano in his book, *The Gambling Scholar*, written in the early 1500s, considers the following carnival game. There are six dice. Each of the dice has five blank sides. The sixth side has a number between 1 and 6 — a different number on each die. The six dice are rolled and the player wins a prize depending on the total of the numbers which turn up. Find the expected total of the numbers.
3. (Section 6.1, Problem 6) A die is rolled twice. Let X denote the sum of the two numbers that turn up, and Y the difference of the numbers (specifically, the number on the first roll minus the number on the second). Show that $E(XY) = E(X)E(Y)$. Are X and Y independent?
4. (Section 6.1, Problem 18) Exactly one of six similar keys opens a certain door. If you try the keys, one after another, what is the expected number of keys that you will have to try before success? (*Note*: This is not a Bernoulli trial, since we are not replacing the keys.)
5. You roll a die until every number shows up. Let R be the number of rolls before every number shows up. Find $E(R)$. (*Hint*: Write $R = R_1 + R_2 + \dots + R_6$, where R_i is the number of rolls until the i^{th} distinct number shows up, but after the $(i - 1)^{\text{st}}$ distinct number shows up.)

(continued on next page)

6. (Section 6.2, Problem 5) In a certain manufacturing process, the (Fahrenheit) temperature never varies by more than 2° from 62° . The temperature is, in fact, a random variable F with distribution

$$P_F = \begin{pmatrix} 60 & 61 & 62 & 63 & 64 \\ 1/10 & 2/10 & 4/10 & 2/10 & 1/10 \end{pmatrix}$$

- (a) Find $E(F)$ and $V(F)$.
- (b) Define $T = F - 62$. Find $E(T)$ and $V(T)$, and compare these answers with those in part (a).
- (c) It is decided to report the temperature readings on a Celsius scale, that is, $C = (5/9)(F - 32)$. What are the expected value and variance for the readings now?
7. For this problem, use the Life Table in Appendix C of the book, p. 501.

A 40-year-old male will buy a life insurance policy that covers the next 4 years. He will pay K dollars on the days he turns 40, 41, 42, and 43, if he is still alive. His family will get \$100,000 if he dies before age 44. Assuming that the insurance company wants an expected profit of \$1000 for this plan, what should K be? (No, there is no interest in this problem!)

Practice problems NOT to turn in: 6.1.2, 6.1.6, 6.1.8, 6.1.19, 6.1.36, 6.2.3, and the following problem:

Practice 2. Find the expected number of aces in a five-card poker hand. Also find the variance.