Homework 5 - Due October 17, 2012

1. Complete Problem 18 in Probability Online. (This will be worth 10 points).

2. (Section 6.2, Problem 4) X is a random variable with $E(X) = 100$ and $V(X) = 15$. Find:

   (a) $E(X^2)$.

   (b) $E(3X + 10)$.

   (c) $E(-X)$.

   (d) $V(-X)$.

   (e) $D(-X)$.

3. If X and Y are any two random variables, then the *covariance* of X and Y is defined by: $\text{Cov}(X, Y) = E((X - E(X))(Y - E(Y)))$. Covariance measures how much X and Y change together.

   (a) What is $\text{Cov}(X, X)$?

   (b) Show that the formula for $\text{Cov}(X, Y)$ simplifies to $E(XY) - E(X)E(Y)$.

   (c) If X and Y are independent, what is $\text{Cov}(X, Y)$?

4. (Section 5.1, Problem 16) Assume that, during each second, a Dartmouth switchboard receives one call with probability .01 and no calls with probability .99. Use the Poisson approximation to estimate the probability that the operator will miss at most one call if she takes a 5-minute coffee break.

5. (Section 5.1, Problem 18) A baker blends 600 raisins and 400 chocolate chips into a dough mix and, from this, makes 500 cookies.

   (a) Find the probability that a randomly picked cookie will have no raisins.

   (b) Find the probability that a randomly picked cookie will have exactly two chocolate chips.

   (c) Find the probability that a randomly picked cookie will have at least two bits (raisins or chips) in it.

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6. Suppose that you draw a single card from a deck repeatedly with replacement (so the card deck is shuffled after every draw).

   (a) Let X be the number of draws until you get your fourth king. What are $E(X)$ and $V(X)$?

   (b) Let Y be the number of kings you draw in 40 draws. What are $E(Y)$ and $V(Y)$?

   (c) Let Z be the number of draws until you get your first king. What are $E(Z)$ and $V(Z)$?

7. (Section 5.1, Problem 26) Feller discusses the statistics of flying bomb hits in an area in the south of London during the Second World War. The area in question was divided into $24 \times 24 = 576$ small areas. The total number of hits was 537. There were 229 squares with 0 hits, 211 with 1 hit, 93 with 2 hits, 35 with 3 hits, 7 with 4 hits, and 1 with 5 or more. Assuming that the hits were purely random, use the Poisson approximation to find the probability that a particular square would have exactly k hits. Compute the expcted number of squares that would have 0, 1, 2, 3, 4, and 5 or more hits and compare this with the observed results.

Practice problems NOT to turn in: 5.1.9 (a)-(b), 5.1.14 (Do this both exactly and with a Poisson approximation), 5.1.20, 5.1.24, 5.1.38, 6.2.9, 6.2.12.