Homework 6 - Due October 24, 2012

1. (Section 5.1, Problem 38) A manufactured lot of buggy whips has 20 items, of which 5 are defective. A random sample of 5 items is chosen to be inspected. Find the probability that the sample contains exactly one defective item

   (a) if the sampling is done with replacement.

   (b) if the sampling is done without replacement.

2. (Section 5.1, Problem 39) Suppose that N and k tend to ∞ in such a way that k/N remains fixed. Show that $h(N, k, n, x) \to b(n, k/N, x)$; that is, show that the hypergeometric distribution tends to the binomial distribution with parameters n and k/N.

3. Suppose that the average number of power failures in Hanover is 2 per month.

   (a) Use Markov's inequality to bound the probability that at least 5 power failures occur this November.

   (b) Assuming that the accidents follow a Poisson distribution, calculate the actual probability that at least 5 power failures occur this November. Compare the two values.

4. (a) Use Chebyshev's inequality to find a lower bound for the probability that the proportion of heads is between 0.4 and 0.6 when you flip a coin 20 times. Do this again for when you flip a coin 100 times and when you flip a coin 1000 times. (So you should have three different answers.)

   (b) Use coinTosses.m from Homework 2 (it may be called coinTosses1.m) to simulate these scenarios 2000 times. What does the actual probablity appear to be for each scenario (when you flip 20 times, 100 times, 1000 times)? Does the answer agree with part (a)? How good of an approximation are the inequalities from part (a)?

   (c) In part (b), why do we need to simulate this scenario 2000 times? Why is the answer we get from our Matlab code close to the actual probability of getting a proportion between 0.4 and 0.6? (Hint: Think of Bernoulli trials. How is each of these 2000 trials a Bernoulli trial?)

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5. (a) Similarly to part (a) of the previous problem, use Chebyshev's inequality to find a lower bound for the probability that the proportion of clubs is between 0.2 and 0.3 when you draw a card from a deck 20 times. Do this again for when you draw from a deck 100 times and 1000 times.

   (b) Similarly to part (b) of the previous problem, simulate these scenarios 2000 times. What does the actual probability appear to be for each scenario? Does this answer agree with part (a)? Note that for this problem, you don't have to start from scratch to write a Matlab program that simulates these scenarios. You should only need to make a few edits to coinTosses.m.

6. You are given the code cointossesLLN.m (on the Syllabus page on the course website). Edit this code so that it returns the average difference between the number of heads and the expected number of heads when you flip n coins *sims* times. So for example, cointossesLLN(10,1000) should flip 10 coins, record the difference between the actual number of heads and the expected number of heads (here you should use "abs" for absolute value of the difference), repeat this experiment 1000 times and return the average (absolute value of the) difference between the number of heads and the expected number of heads.

   (a) What is the average difference when $n = 10$? What about $n = 100$? $n = 1000$? Use 2000 simulations for each.

   (b) Does the average difference increase or decrease as n increases? Does this contradict the Law of Large Numbers?

   (c) Upload your code to Probability Online (Problem 19), and input your answer to (a) for $n = 1000$.

Practice problems NOT to turn in: 8.1.1, 8.1.6, 8.1.7, 8.1.9, 8.1.10, 8.1.11