Homework 8 - Due November 13, 2012 at 4 PM

1. (Section 11.1, Problem 2) In Example 11.4, let $a = 0$ and $b = 1/2$. Find $\mathbf{P}, \mathbf{P^2}$, and $\mathbf{P^3}$. What would $\mathbf{P^n}$ be? What happens to $\mathbf{P^n}$ as n tends to infinity? Interpret this result.

2. (Section 11.2, Problem 18) Assume that a student going to a certain four-year medical school in northern New England has, each year, a probability q of flunking out, a probability r of having to repeat the year, and a probability p of moving on to the next year (in the fourth year, moving on means graduating).

   (a) Form a transition matrix for this process taking as states F, 1, 2, 3, 4, and G where F stands for flunking out and G for graduating, and the other states represent the year of study.

   (b) For the case $q = .1$, $r = .2$, and $p = .7$ find the time a beginning student can expect to be in the second year. How long should this student expect to be in medical school?

   (c) Find the probability that this beginning student will graduate.

3. Complete Probability Online, Problem 34. This will be similar, but not quite equal, to Section 11.2, Problem 23. Upload your code and enter your answer.

4. (Section 11.2, Problem 13) Smith is in jail and has 3 dollars; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. If Smith bets A dollars, he wins A dollars with probability 0.4 and loses A dollars with probability 0.6. Find the probability that he wins 8 dollars before losing all of his money if

   (a) he bets 1 dollar each time (timid strategy). *Note*: Your program from problem 3 should be able to compute this answer.

   (b) he bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars (bold stategy). *Note*: The answer in the odd-numbered solutions manual is incorrect.

   (c) Which strategy gives Smith the better chance of getting out of jail?

(continued on next page)

5. (Section 11.3, Problem 3) Consider Example 11.4 again, where the transition matrix is

$$P = \begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}$$

   (a) Under what conditions is P absorbing?
   (b) Under what conditions is P ergodic but not regular?
   (c) Under what conditions is P regular?

6. (Section 11.3, Problem 4) Find the fixed probability vector $\mathbf{w}$ for the matrices in Problem 5 that are ergodic.

Problems 7-9 are 3 points each.

7. (Section 11.3, Problem 14) Consider an independent trials process to be a Markov chain whose states are the possible outcomes of the individual trials. What is its fixed probability vector? Is the chain always regular? Illustrate this for Example 11.5.
8. (Section 11.3, Problem 28) Prove that $\mathbf{P}$ and $(1/2)(\mathbf{I}+\mathbf{P})$ have the same fixed vectors.
9. Give an example of a Markov chain that is neither absorbing nor ergodic. Write down the transition matrix for this Markov chain. Justify your answer.
10. (Extra Credit) Complete Probability Online, Problem 35.

Practice problems NOT to turn in: 11.1.4, 11.1.5, 11.1.6, 11.1.11, 11.2.6, 11.2.7, 11.2.12, 11.2.14, 11.3.1, 11.3.12, 11.3.24, 11.3.26, 11.3.27