

Math 22, Summer 2013, Midterm I

Name (Print):

Last

First

Instructions: You are not allowed to use calculators, books, or notes of any kind. You may not look at a classmate's exam for "inspiration." You must explain your reasoning behind each solution to receive full credit. Credit will not be awarded for correct answers with no explanation (with the exception of problem #1).

You may use pages 11-14 of the exam as scratch paper, but any work you intend to be graded should be on the exam itself in the space provided. If you run out of room, clearly indicate which page of scratch paper your solution is on and circle the solution that should be graded.

Before beginning the exam, skim through the problems to verify that you have one true/false question, four free-response questions, and one bonus question.

| Problem # | Points | Score |
|--------------|----------------|-------|
| 1 | 5 | |
| 2 | 13 | |
| 3 | 16 | |
| 4 | 8 | |
| 5 | 8 | |
| BONUS | 3 | |
| Total | 50 (w/o bonus) | |

1. Determine whether each statement below is true or false and indicate your answer by circling the appropriate choice (1pt each):
- (a) (True / False) If A is a 3×3 matrix with 2 pivot columns, then $A\mathbf{x} = \mathbf{b}$ has a solution for each \mathbf{b} in \mathbb{R}^3 .
- (b) (True / False) Suppose $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are vectors in \mathbb{R}^n , and \mathbf{b} is also a vector in \mathbb{R}^n . Then \mathbf{b} is in $Span\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ if and only if $[\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \mathbf{b}$ has a solution.
- (c) (True / False) The columns of a 2×3 matrix A must be linearly dependent.
- (d) (True / False) The matrix $A = \begin{bmatrix} 4 & -h \\ h & 1 \end{bmatrix}$ is not invertible for some scalar h .
- (e) (True / False) If T is a linear transformation, then T either maps a line with parametric form $\mathbf{x} = \mathbf{p} + t\mathbf{v}$, $t \in \mathbb{R}$, to a line or a point in its codomain.

2. Let $A = \begin{bmatrix} 1 & 3 & 5 & 9 \\ 2 & 4 & 6 & 7 \\ 1 & 2 & 3 & 4 \end{bmatrix}$.

- (a) Let $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$. Determine whether $A\mathbf{x} = \mathbf{b}$ is consistent or inconsistent. If $A\mathbf{x} = \mathbf{b}$ is consistent, describe its solution set in parametric vector form. If the system is inconsistent, explain why. (5pts)

- (b) Is $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ in the span of the columns of A ? Explain. (2pts)

- (c) How many columns of A are pivot columns? (3pts)

(d) Is $A\mathbf{x} = \mathbf{b}$ consistent for every \mathbf{b} in \mathbb{R}^3 ? Explain. (3pts)

3. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$.

- (a) Determine the solution set in parametric vector form of the homogeneous equation $A\mathbf{x} = \mathbf{0}$. (2pts)

- (b) The aim of the following statement is to provide a geometric description of the solution set found in part (a). Fill in the blanks to make the following statement true (2pts):

“The solution set of $A\mathbf{x} = \mathbf{0}$ obtained in part (a) is a _____ in \mathbb{R}^3 through _____.”

- (c) Are the columns of A linearly independent? Explain. (2pts)

- (d) Let T be the transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(\mathbf{x}) = A\mathbf{x}$ for all \mathbf{x} in \mathbb{R}^3 , where A is the 3×3 matrix given at the beginning of the problem. Provide an explanation when answering the following 3 questions (2 pts each):

i. Is T linear?

ii. Is T onto \mathbb{R}^3 ?

iii. Is T one-to-one?

(e) Let $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Is $A\mathbf{x} = \mathbf{b}$ consistent? Explain. (2pts)

(f) Let $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Describe geometrically the solution set of $A\mathbf{x} = \mathbf{b}$ in relation to the solution set of the homogeneous equation $A\mathbf{x} = \mathbf{0}$. (2pts)

4. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

(a) Compute the determinant of A . (3pts)

(b) Use elementary row operations to compute the inverse of A . (5pts)

5. Determine the standard matrix A of the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that first maps the unit vector \mathbf{e}_1 to itself and maps the unit vector \mathbf{e}_2 to the vector $2\mathbf{e}_1 + \mathbf{e}_2$, followed by a reflection across the line $x_2 = x_1$. Explain your work to receive full credit. (8 pts)

6. **BONUS PROBLEM** Suppose $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a linearly independent set in \mathbb{R}^n . Prove that $\{\mathbf{v}_1, \mathbf{v}_1 + \mathbf{v}_2\}$ is also linearly independent.

