Barnett
11/16/05 &
11/26/07

MATH23 WORKSHEET : Even & odd Fourier series.

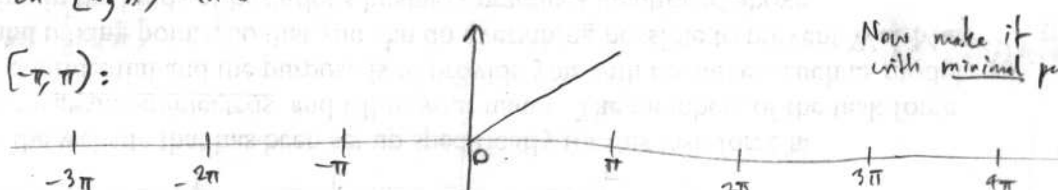
Even } functions have { $f(-x) = f(x)$
Odd } functions have { $f(-x) = -f(x)$

Fill in the table [hint: sketch!]

If $f(x)$ is	and $g(x)$ is	then $h(x) = f(x)g(x)$ is
even	even	☐
odd	even	☐
odd	odd	☐

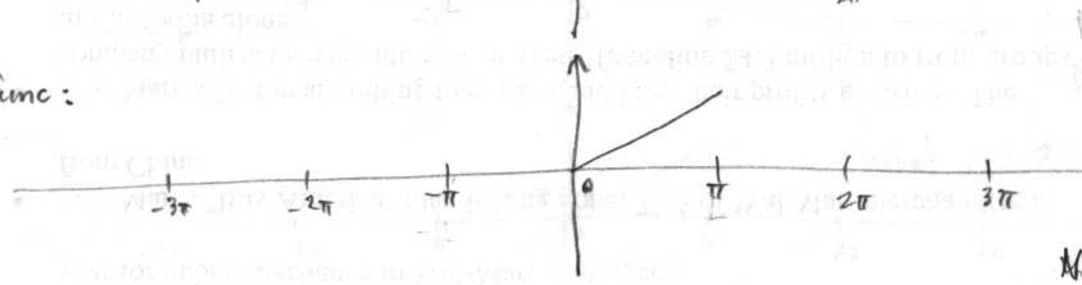
Is $\cos\frac{n\pi x}{L}$ / $\sin\frac{n\pi x}{L}$ even or odd? { ☐ / ☐ } ← How about $n=0$?

Extend $f(x) = x$ on $[0,\pi)$ into an <u>even</u> func in $[-\pi,\pi)$:



Now make it periodic with <u>minimal</u> period. What is L?

or an <u>odd</u> func:



Now make it periodic with same L. Is it continuous?

When $f(x)$ is { even , / odd , what is { $\int_{-\pi}^{\pi} f(x)\sin nx\,dx$? / $\int_{-\pi}^{\pi} f(x)\cos nx\,dx$? (Hint: no integral required!)

When $f(x)$ is even, find the Fourier coefficients $a_0, a_1, \dots$ $b_1, b_2 \dots$ (a_0: easy; $a_1,\dots$: harder)

Hint:
$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$
$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$

PTO for workspace

~ SOLUTIONS ~

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MATH23 WORKSHEET: Even & odd Fourier series.

Even } functions have { $f(-x) = f(x)$
Odd }	$f(-x) = -f(x)$

Fill in the table [hint: sketch!]

If $f(x)$ is	and $g(x)$ is	then $h(x) = f(x)g(x)$ is
even	even	even
odd	even	odd
odd	odd	even

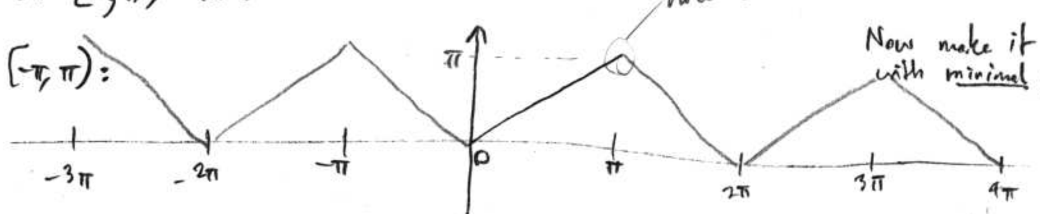
check it! Eg x is odd, x^2 is even

Is $\begin{cases} \cos \frac{n\pi x}{L} \\ \sin \frac{n\pi x}{L} \end{cases}$ even or odd? $\begin{cases} \text{even} \\ \text{odd} \end{cases}$ ← How about $n=0$? (also even).

Extend $f(x) = x$ on $[0,\pi)$ into

an even func in $[-\pi,\pi)$:

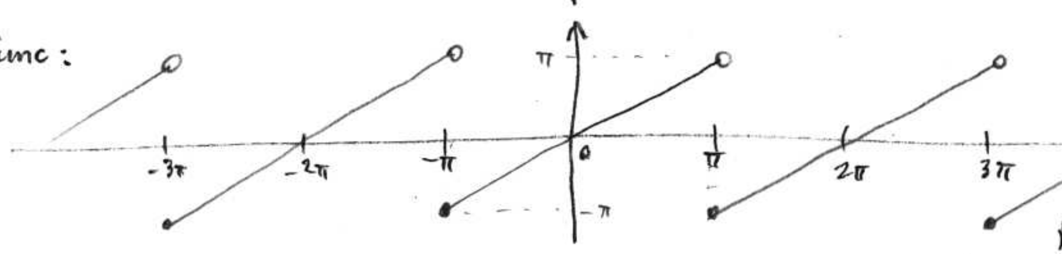
note: continuous



Now make it periodic with minimal period. What is L?

$L = \pi$.

or an odd func:



$L = \pi$

Now make it periodic with same L.

Is it continuous? No: jump at $x = \pi$

When $f(x)$ is $\begin{cases} \text{even}, \\ \text{odd}, \end{cases}$ what is $\begin{cases} \int_{-\pi}^{\pi} f(x) \sin nx \, dx \ ? \quad 0 \\ \int_{-\pi}^{\pi} f(x) \cos nx \, dx \ ? \quad 0 \end{cases}$ ← both are integrals of (odd)(even) ie an odd func.

Hint: no integral required!

When $f(x)$ is even, find the Fourier coefficients $a_0, a_1, \ldots$ (easy?) $b_1, b_2 \ldots$ (harder)

Hint:
$a_n = \frac{1}{L}\int_{-L}^{L} f(x) \cos\frac{n\pi x}{L} dx$
$b_n = \frac{1}{L}\int_{-L}^{L} f(x) \sin\frac{n\pi x}{L} dx$

immediately $b_n = 0$ for all n using the above.

PTO for workspace

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} |x| \, dx = \frac{2}{\pi}\cdot\frac{\pi^2}{2} = \pi$ ← indeed is twice the avg. value of f.

$a_n = \frac{2}{\pi}\int_0^{\pi} x \cos nx \, dx \overset{\text{by parts}}{=} -\frac{2}{\pi}\frac{1}{n}\int_0^{\pi} \sin nx \, dx = \frac{-4}{\pi n^2}$ for n odd, zero otherwise. $(n>0)$