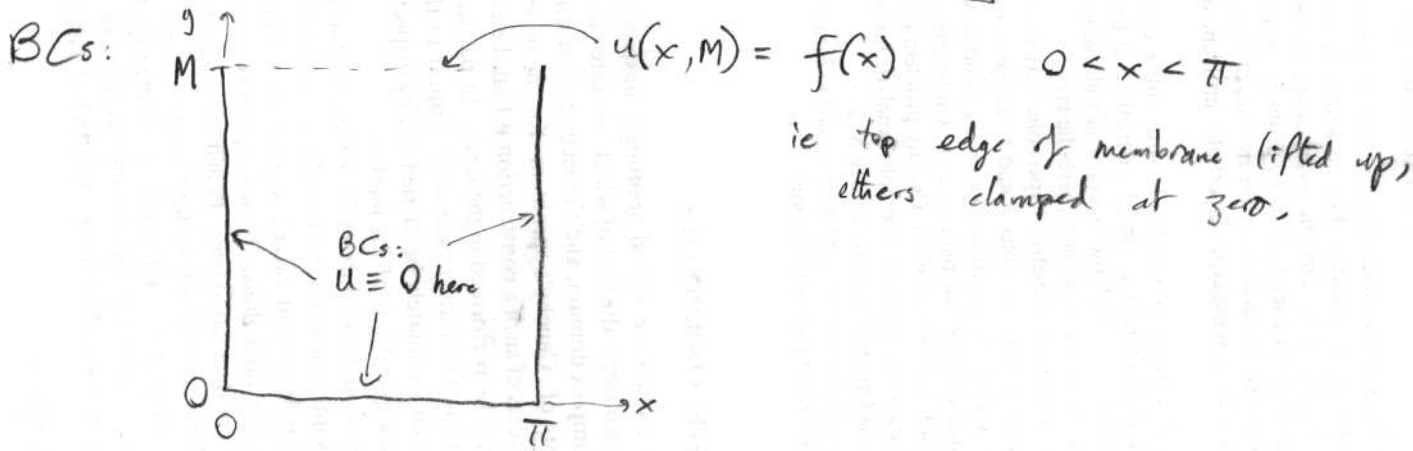


PDE: $U_{xx} + U_{yy} = 0$



Separation of variables: Hint: $u(x, y) = X(x) Y(y)$ & separate:

(choose a sep. const. making $X(x)$ as before)

HORIZONTAL:
 $X(x)$: write 2-pt BVP, find eigenfuncs, X_n ,
eigenvalues λ_n

VERTICAL:
Now use your λ_n to write 2-pt BVP
for $Y_n(y)$:

is $Y_n(y)$ oscillatory or decaying/growing?

Use bottom BC $u(x, 0) = 0 \quad 0 < x < \pi$ to
narrow down the form of $Y_n(y)$:

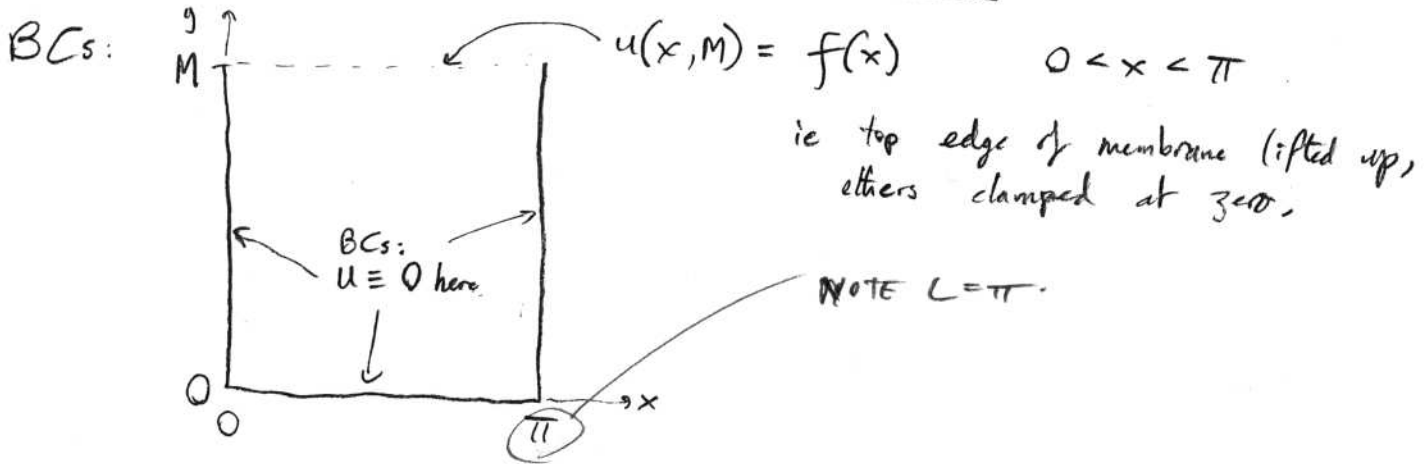
Write a general solution: coeffs.

$$u(x, y) = \sum_{n=1}^{\infty} c_n \left(\begin{array}{c} \text{nth func. of } x \end{array} \right) \cdot \left(\begin{array}{c} \text{nth func. of } y \end{array} \right)$$

Solve for coeffs c_n in terms of b_n , the Fourier sine coeffs of f :

~ SOLUTIONS ~

PDE: $U_{xx} + U_{yy} = 0$



Separation of variables:

Hint: $u(x, y) = X(x) Y(y)$ & separate:

PDE: $X''Y = -XY''$

$$\frac{X''}{X} = -\lambda = -\frac{Y''}{Y}$$

(choose a sep. const. making $X(x)$ as before)

HORIZONTAL:

$X(x)$: write 2-pt BVP, find eigenforms X_n , eigenvalues λ_n

$$\left. \begin{aligned} X'' + \lambda X &= 0 \\ X(0) = X(\pi) &= 0 \end{aligned} \right\} \text{2pt BVP}$$

eigenforms $X_n(x) = \sin \frac{n\pi x}{L} = \sin nx$

$$\lambda_n = \frac{\pi^2 n^2}{L^2} = n^2$$

$$n = 1, 2, \dots$$

VERTICAL:

Now use your λ_n to write 2pt BVP for $Y_n(y)$:

$$Y'' - \lambda Y = 0$$

note sign λ implies.

is $Y_n(y)$ oscillatory or decaying/growing?

Use bottom BC $u(x, 0) = 0 \quad 0 < x < \pi$ to narrow down the form of $Y_n(y)$:

$$Y_n(y) = A e^{+\sqrt{\lambda_n} y} + B e^{-\sqrt{\lambda_n} y}$$

$$Y_n(0) = 0 \quad \text{so } B = -A, \text{ so } Y_n(y) = e^{ny} - e^{-ny}$$

Write a general solution:

$$u(x, y) = \sum_{n=1}^{\infty} C_n \left(\sin nx \right) \cdot \left(e^{ny} - e^{-ny} \right)$$

coeffs.
 n^{th} func. of x
 n^{th} func. of y
 also = $2 \sinh ny$

Solve for coeffs C_n in terms of b_n , the Fourier sine coeffs of f :
since $u(x, M) = f(x) = \sum_{n=1}^{\infty} b_n \sin nx$, equating coeffs to the above.

$$C_n = \frac{b_n}{e^{nM} - e^{-nM}}$$