9/26/07, 9/21/05

Math 23 WORKSHEET on const-coeff linear 1^{st} order.

Solution of $y' = ay - b$

Repeat the algebra used for 'mice & owls' to write general solution:
(with integration const. c)

Think about large times $t \to \infty$:

i) What condition on a, b, c gives decay (stable behavior)?

ii) What gives growth $y \to +\infty$?

ii) What happens if $a > 0$, $c = 0$?

9/26/07, 9/21/05

Math 23 WORKSHEET on const-coeff linear 1st order.

Solution of $y' = ay - b$

Repeat the algebra used for 'mice & owls' to write general solution (with integration const. c)

$$y' = a(y - b/a)$$

$$\frac{y'}{y - b/a} = a$$

integrate w.r.t. t
$$\ln|y - b/a| = at + c' \Rightarrow |y - b/a| = e^{at}e^{c'}$$

$$\Rightarrow y - b/a = ce^{at} \Rightarrow y(t) = b/a + ce^{at}$$

Think about large times $t \to \infty$:

i) What condition on a, b, c gives decay (stable behavior)?

$a < 0$ gives exponential decay to b/a, no matter what c is!

ii) What gives growth $y \to +\infty$?

$a > 0$ $c > 0$ (otherwise $c < 0$ goes to $-\infty$)

ii) What happens if $a > 0$, $c = 0$?

e^{at} grows without limit, but this is killed by $c = 0$

so $y = b/a$ constant for all time.