MATH 23 WORKSHEET : Row reduction.

Barnett
10/24/05

To invert $A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 0 & 1 \\ -3 & 1 & 2 \end{bmatrix}$ write I next to it & perform row operations, to get $A \to I$

> i) multiply a row by a number
> ii) add multiple of row to another row
> iii) Swap 2 rows.

Start.

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 & 1 & 0 \\ -3 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$\overbrace{}^{I}$

how much of R_1 to add to cancel the -2?

same for the -3 in R_3.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & & & & & \\ 0 & & & & & \end{array}\right]$$

Fill in rest., including RHS!

now make 2nd entry in R_3 go away by adding / subtracting R_2 from it.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & & & & & \\ 0 & 0 & & & & \end{array}\right]$$

make sure all diag. entries on LHS are 1.

Now add some of R_3 to R_2 to remove 3rd entry on R_2:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & & \end{array}\right]$$

Now make the -1 go away here.

$$\left[\begin{array}{ccc|ccc} 1 & \textcircled{0} & 0 & & & \\ 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & & \end{array}\right]$$

↑ Here you'll find A^{-1}.

Can check by multiplying $A^{-1}A$ or AA^{-1} !

MATH 23 WORKSHEET : Row reduction for inverse.

Barnett
10/27/05

~ SOLUTIONS ~

To invert $A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 0 & 1 \\ -3 & 1 & 2 \end{bmatrix}$ write I next to it & perform row operations, to get $A \to I$

i) multiply a row by a number
ii) add multiple of row to another row
iii) Swap 2 rows.

Start.

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 & 1 & 0 \\ -3 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

(the right block is $\overbrace{\quad}^{I}$)

how much of R_1 to add to cancel the -2?

Same for the -3 in R_3.

$$\begin{array}{c} \\ R_2 + 2R_1 \\ R_3 + 3R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 2 & 1 & 0 \\ 0 & -2 & 2 & 3 & 0 & 1 \end{array}\right]$$

Fill in rest., including RHS!

now make 2^{nd} entry in R_3 go away by adding / subtracting R_2 from it.

$$\begin{array}{c} \\ \\ R_3 - R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right]$$

make sure all diag. entries on LHS are 1.
Now add some of R_3 to R_2 to remove 3^{rd} entry on R_2:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & -1 & 1/2 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right] \leftarrow -\tfrac{1}{2}(R_2 - R_1)$$

rescale (multiply) R_2 by $-1/2$ <u>after</u> you subract R_3 easi
(tricky, you could do it in two stages.)

Now make the -1 go away here.

$$\left[\begin{array}{ccc|ccc} 1 & \boxed{0} & 0 & 1/2 & -1 & 1/2 \\ 0 & 1 & 0 & -1/2 & -1 & 1/2 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right] \quad R_1 + R_2$$

I $\qquad$ A^{-1} ↑ Here you'll find A^{-1}.

Can check by multiplying $A^{-1}A$ or AA^{-1} !