MATH 23 WORKSHEET : Homog const. coeff lin sys. Barnett 11/8/07

A) Find general solution to $\vec{x}' = A\vec{x}$ for $A = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix}$

Find eigvals:

Find corresp. eigenvecs:

Write $\vec{x}(t) = c_1 \vec{\xi}^{(1)} e^{r_1 t} + c_2 \vec{\xi}^{(2)} e^{r_2 t}$:

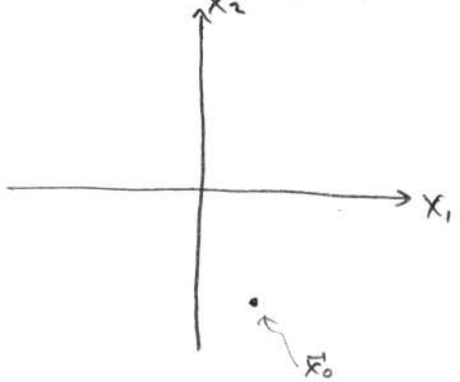
$\vec{x}(t) =$

B) Match the initial condition $\vec{x}(0) = \vec{x}_0 = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

[Hint: solve c_1, c_2 via row reduction]

So, solution is : $\vec{x}(t) = \ldots$

C) Sketch behavior of solutions in (x_1, x_2) plane:



[include a variety of ICs, show flow directions, relate to eigenvectors (sketch them first!)].

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~ SOLUTIONS ~

A) Find general solution to $\vec{x}' = A\vec{x}$ for $A = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix}$

Find eigvals: $\begin{vmatrix} 1-\lambda & 4 \\ 1 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - (1)4$

so $\lambda^2 - 2\lambda + 1 - 4 = 0$

$(\lambda - 3)(\lambda + 1) = 0$

$\lambda = -1, +3.$

distinct so <u>will</u> get full set of evecs.

Find corresp. eigenvecs:

$\boxed{\lambda = -1:}$ $(A - -1I)\vec{v} = \begin{bmatrix} 1+1 & 4 \\ 1 & 1+1 \end{bmatrix}\vec{v} = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}\vec{v} =$

so $\vec{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\boxed{\lambda = +3:}$ $\begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix}\vec{v} = \vec{0}$ so $\vec{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Write $\vec{x}(t) = c_1 \vec{\xi}^{(1)} e^{r_1 t} + c_2 \vec{\xi}^{(2)} e^{r_2 t}$:

$$\vec{x}(t) = c_1 \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t}$$

B) Match the initial condition $\vec{x}(0) = \vec{x}_0 = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

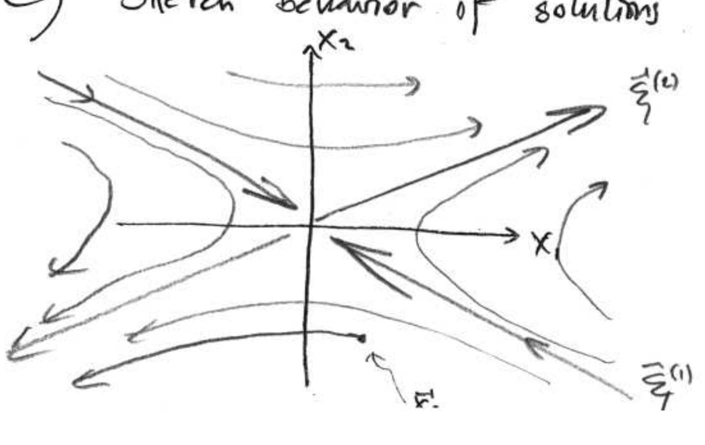
[Hint: solve c_1, c_2 via row reduction]

$c_1 \vec{\xi}^{(1)} + c_2 \vec{\xi}^{(2)} = \vec{x}_0$ ie $\left[\begin{array}{cc|c} 2 & 2 & 2 \\ -1 & 1 & -5 \end{array}\right]$ is linear system

$\sim \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 2 & -4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array}\right]$ so $c_1 = 3$, $c_2 = -2$

So, solution is: $\vec{x}(t) = 3\begin{pmatrix} 2 \\ -1 \end{pmatrix} e^{-t} - 2\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t}$

C) Sketch behavior of solutions in (x_1, x_2) plane:



[include a variety of ICs, show flow directions, relate to <u>eigenvectors</u> (sketch them first!)].

saddle point (hyperbolic flow curves).