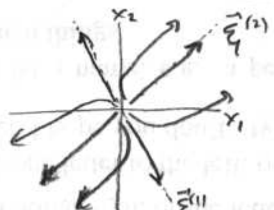
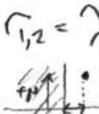
MATH 23 WORKSHEET : Summarizing stability of $\underline{\text{lin.}}$ sys.

Barnett 11/14/07.

Imagine the eigenvalues of $\vec{x}' = A\vec{x}$ are r_1, r_2

(A: 2×2 matrix)

Complete the table:

case	type of critical pt.	Sketch phase plane	AS?	Stable? (S)
$0 < r_1 < r_2$ (real eigvals)	nodal source		X	X
$r_1 < r_2 < 0$				
$r_1 < 0 < r_2$				
$r_{1,2} = \lambda \pm i\mu$, $\lambda > 0$ 				
$\lambda < 0$				
$\lambda = 0$				
$r_1 = r_2$ > 0	proper node or improper node (if not both eigvecs present)			
< 0		sketch the improper case.		

Summary: (fill in the blanks!) if the maximum $\boxed{}$ part of $r_{1,2}$ is $\boxed{}$ then : S & AS.

or is $\boxed{}$ then : S but $\underline{\text{not}}$ AS.

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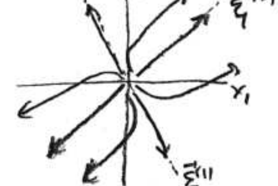
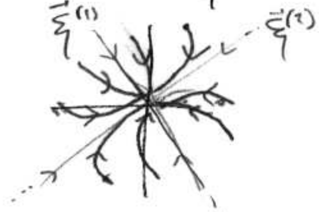
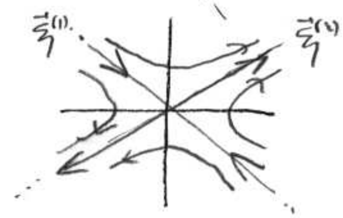
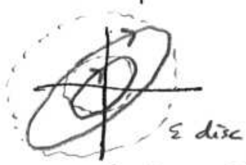
— SOLUTIONS ~.

Imagine the eigenvalues of $\vec{x}' = A\vec{x}$ are r_1, r_2.

(A: 2x2 matrix.)

Complete the table:

* Note $AS \Rightarrow S$ but <u>not</u> other way round

case	type of critical pt.	Sketch phase plane	AS?	Stable? (S)
$0 < r_1 < r_2$ (real eigvals)	nodal source		×	×
$r_1 < r_2 < 0$	nodal sink		✓	✓
$r_1 < 0 < r_2$	saddle pt.		× in ~~every~~ disc about origin there are points which go off to ∞ as $t \to \infty$	×
$r_{1,2} = \lambda \pm i\mu$, $\lambda > 0$	spiral source		×	×
$\lambda < 0$	spiral sink		✓	✓
$\lambda = 0$	center		× since don't tend to origin.	✓ since stay within an ε disc
$r_1 = r_2 > 0$	proper node or improper node (if not both eigvecs present)		×	×
< 0		 sketch the improper case.	✓	✓

Summary: (fill in the blanks!) if the maximum $\boxed{\text{real}}$ part of $r_{1,2}$ is $\boxed{>0}$ then : S & AS.

or is $\boxed{=0}$ then : S but not AS.

so S given by $\max \operatorname{Re} r \le 0$