

Recall $y'' + y = 0$ has general soln. $y(x) = A \cos x + B \sin x$

Write down the equations for A, B given by substituting for x at the 2 ends.

Use this to determine existence, uniqueness.

Solve simultaneous eqns:

i)

eqn here is $A \cos 0 + B \sin 0 = 0$
 $L = ?$

eqn here is: $= 2$

A	B	exists?	unique?

ii)

eqn here is $\boxed{=}$
 $\boxed{=}$

first write out general $y'(x) = \dots$

solve:

A	B	exist?	unique?

if exists, sketch $y(x)$ in interval.

iii)

eqn here is $\boxed{=}$
 $\boxed{=}$

as before

A	B	exist?	unique?

sketch?

to test, try doubling A & B to see if also a solution!

iv)

eqn here is $\boxed{=}$
 $\boxed{=}$

			unique?

think carefully here!

Consider specifying 2 values (ie y not y'), ie $y(0)$ & $y(L)$, as in i) or iv).

* For what values of L is there nonexistence/nonuniqueness?

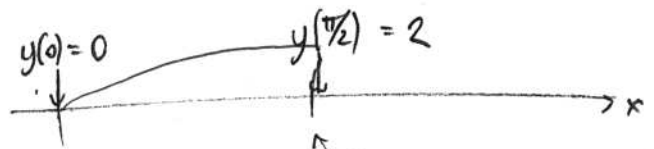
* At these values of L , what is condition on $y(0), y(L)$ for existence?

~ SOLUTIONS ~

Recall $y'' + y = 0$ has general soln. $y(x) = A \cos x + B \sin x$

Write down the equations for A, B given by substituting for x at the 2 ends.

Use this to determine existence, uniqueness.

i) 

eqn here is $A \cos 0 + B \sin 0 = 0$
 $A = 0$

eqn here is $A \cos \pi/2 + B \sin \pi/2 = 2$
 $B = 2$

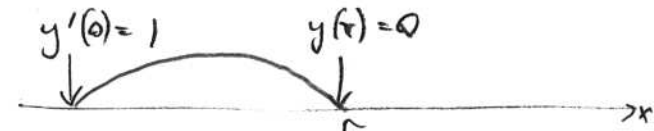
Solve simultaneous eqns:

A	B	exists?	unique?
0	2	✓	✓

ie $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

det $\neq 0$ so exists & unique

first write out general $y'(x) = -A \sin x + B \cos x$

ii) 

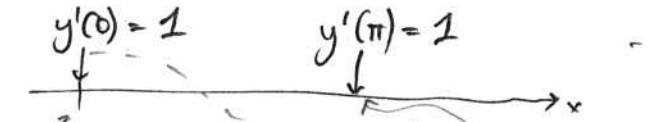
eqn here is $-A \sin 0 + B \cos 0 = 1$
 $-A \cos \pi + B \sin \pi = 0$

ie $A = 1, B = 0$

solve:

A	B	exist?	unique?
0	1	✓	✓

if exists, sketch $y(x)$ in interval.

iii) 

$-A \sin 0 + B \cos 0 = 1$
 $B = 1$

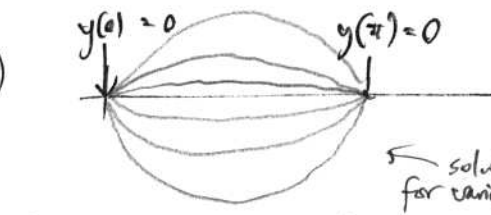
$-A \sin \pi + B \cos \pi = 1$
 $B = -1$

inconsistent in eqns!

A	B	exist?	unique?
		✗	

sketch?

to test, try doubling A & B to see if also a solution!

iv) 

$A \cos 0 + B \sin 0 = 0$
 $A \cos \pi + B \sin \pi = 0$

ie both eqns say $A = 0$
 But $B = \text{anything}$

solutions for various B .

0	anything	✓	unique?
			✗

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ ~ lin. sys.

think carefully here!

Consider specifying 2 values (ie y not y'), ie $y(0)$ & $y(L)$, as in i) or iv).

* For what values of L is there nonexistence/nonuniqueness? $L = n\pi$, since there $\sin L = \sin 0 = 0$

* At these values of L , what is condition on $y(0), y(L)$ for existence? since otherwise the 2 eqns for A incons. $y(L) = (-1)^n y(0)$