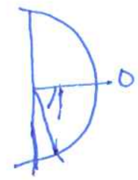
Worksheet #11



(1) Determine ω_0, R, and δ so as to write

$$u = 4\cos 3t - 2\sin 3t$$

of the ~~from~~ form $u = R\cos(\omega_0 t - \delta)$.

$4 = R\cos\delta \qquad -2 = R\sin\delta \to R^2 = 16 + 4 = 20 \Rightarrow R = 2\sqrt{5}$

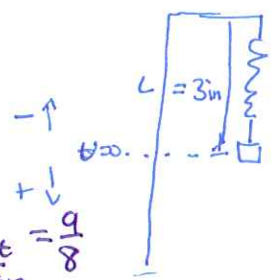
$-\frac{1}{2} = \frac{-2}{4} = \frac{\sin\delta}{\cos\delta} = \tan\delta \to \delta = \arctan(-1/2)$

$u(t) = 2\sqrt{5}\cos(3t + \delta)$

(2) A mass weighing 3 lbs stretches a spring 3 in. If the mass is pushed upward, contracting the spring a distance of 1 in. and then set in motion with a downward velocity of 2 ft/s, and if there is no damping, find the position u of the mass at any time t. Determine the frequency, period, amplitude, and phase of the motion.

$mg = 3\text{ lb}$

$\Rightarrow m = \frac{3\text{lb}}{g} = \frac{3\text{lb}}{32\text{ lb s}^2/\text{ft}} = \frac{3\text{ lb}}{32\text{ lb s}^2/\text{ft}} \cdot \frac{1\text{ft}}{12\text{in}} = \frac{9}{8}$



$L = \frac{1}{4}\text{ft} = 3\text{in}$

We know $mg - kL = 0 \Rightarrow k = \frac{mg}{L} = \frac{3}{3\text{in}} = 1\ \frac{\text{lb}}{\text{in}}$ = spring constant

No damping

Initial conditions $\begin{cases} u(0) = -1 \\ u'(0) = 2\frac{\text{ft}}{\text{s}} \cdot \frac{12\text{in}}{\text{ft}} = 36\frac{\text{in}}{\text{s}} \end{cases}$ eqn. $\left\{ \frac{9}{8}u'' + u = 0 \to u'' + \frac{8}{9}u = 0 \right.$

1 - Solve IVP.

Characteristic eqn $r^2 + \frac{8}{9} = 0 \to r = \pm i\frac{2\sqrt{2}}{3}$

$\to u(t) = c_1\cos\left(\frac{2\sqrt{2}}{3}t\right) + c_2\sin\left(\frac{2\sqrt{2}}{3}t\right)$

$$u(0) = c_1 = -1$$

$$u'(t) = -\frac{2\sqrt{2}}{3} c_1 \sin\left(\frac{2\sqrt{2}}{3} t\right) + c_2 \frac{2\sqrt{2}}{3} \cos\left(\frac{2\sqrt{2}}{3} t\right)$$

$$u'(0) = c_2 \frac{2\sqrt{2}}{3} = 2 \rightarrow c_2 = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

$$\rightarrow u(t) = -1 \cos\left(\frac{2\sqrt{2}}{3} t\right) + \frac{3\sqrt{2}}{2} \sin\left(\frac{2\sqrt{2}}{3} t\right)$$

$$R^2 = (-1)^2 + \left(\frac{3\sqrt{2}}{2}\right)^2 = 1 + (9) = 10 \rightarrow \boxed{R = 10 = \text{Amplitude}}$$

$$\frac{\frac{3\sqrt{2}}{2}}{-1} = \tan\delta \rightarrow \boxed{\delta = \arctan\left(-3\sqrt{2}/2\right) = \text{phase}}$$

$$\boxed{\text{Period } T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\frac{2\sqrt{2}}{3}} = \frac{\pi\, 3\sqrt{2}}{2}}$$

$$\boxed{\text{frequency } \omega_0 = \frac{2\sqrt{2}}{3}}$$