Worksheet #17

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(1) Show that $A = \begin{bmatrix} 4 & -1 & 3 \\ 2 & 5 & -1 \\ -8 & 2 & -6 \end{bmatrix}$ is singular. ie show $\det(A) = 0$

$$\det(A) = \begin{vmatrix} 4 & -1 & 3 \\ 2 & 5 & -1 \\ -8 & 2 & -6 \end{vmatrix} = 4\begin{vmatrix} 5 & -1 \\ 2 & -6 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ -8 & -6 \end{vmatrix} + 3\begin{vmatrix} 2 & 5 \\ -8 & 2 \end{vmatrix}$$

$$= 4(-30+2) + 1(-12-8) + 3(4+40) = 0$$

(2) Consider the system of equations

$$y + z = 6$$
$$3x - y + z = -7$$
$$x + y + z = -13$$

(a) Rewrite the sytem in matrix form. (ie. $A\boldsymbol{x} = \boldsymbol{b}$)

$$\begin{bmatrix} 0 & 1 & 1 \\ 3 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -7 \\ -13 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13 \\ -7 \\ 6 \end{bmatrix}$$

(b) Solve the system using row reduction.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -13 \\ 3 & -1 & 1 & -7 \\ 0 & 1 & 1 & 6 \end{array}\right] \xrightarrow{②= -3①+②} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -13 \\ 0 & -4 & -2 & 32 \\ 0 & 1 & 1 & 6 \end{array}\right]$$

$$\xrightarrow{②= -\frac{1}{4}②} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -13 \\ 0 & 1 & \frac{1}{2} & -8 \\ 0 & 1 & 1 & 6 \end{array}\right] \xrightarrow{③=-②+③} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -13 \\ 0 & 1 & \frac{1}{2} & -8 \\ 0 & 0 & \frac{1}{2} & 14 \end{array}\right]$$

$$\xrightarrow{③= 2③} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -13 \\ 0 & 1 & \frac{1}{2} & -8 \\ 0 & 0 & \cancel{\frac{1}{2}}\,1 & 28 \end{array}\right]$$

Back solve $\Rightarrow$

$z = 28$

$y = -8 - \frac{1}{2}z = -8 - 14 = -22$

$x = -13 - y - z = -13 + 22 - 28$
$= -13 - 6 = -19$

(b) Find A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \; (2) = -3(1)+(2) \; \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -4 & -2 & -3 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$(2) = -\tfrac{1}{4}(2) \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & 3/4 & -1/4 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{(3) = -(2)+(3)} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & 3/4 & -1/4 & 0 \\ 0 & 0 & 1/2 & -3/4 & +1/4 & 1 \end{array}\right]$$

$$(3) = 2(3) \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & 3/4 & -1/4 & 0 \\ 0 & 0 & 1 & -3/2 & 1/2 & 2 \end{array}\right] \begin{array}{l} (1) = (1) - (3) \\ (2) = (2) - \tfrac{1}{2}(3) \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 5/2 & -1/2 & -2 \\ 0 & 1 & 0 & 3/2 & -1/2 & -1 \\ 0 & 0 & 1 & -3/2 & 1/2 & 2 \end{array}\right]$$

$$(1) = (1) - (2) \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 3/2 & -1/2 & -1 \\ 0 & 0 & 1 & -3/2 & 1/2 & 2 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 3/2 & -1/2 & -1 \\ -3/2 & 1/2 & 2 \end{bmatrix}$$

(c) Verify that $x = A^{-1}b$ is the same as your answer from part (b).

Now $\bar{x} = A^{-1} b$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 3/2 & -1/2 & -1 \\ -3/2 & 1/2 & 2 \end{bmatrix} \begin{bmatrix} -13 \\ -7 \\ 6 \end{bmatrix} = \begin{bmatrix} -13 - 6 \\ 3/2(-13) - 1/2(-7) - 6 \\ -3/2(-13) + -7/2 + 12 \end{bmatrix} = \begin{bmatrix} -19 \\ -22 \\ 28 \end{bmatrix}$$