Worksheet #18

(1) Are the vectors $x = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$, $y = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}$, and $z = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ linearly dependent or independent?

to determine linear independence look at

$$\det\left(\begin{bmatrix} 2 & 3 & 0 \\ -2 & -5 & 1 \\ 4 & 4 & 2 \end{bmatrix}\right) = \begin{vmatrix} 2 & 3 & 0 \\ -2 & -5 & 1 \\ 4 & 4 & 2 \end{vmatrix} = 2\begin{vmatrix} -5 & 1 \\ 4 & 2 \end{vmatrix} - 3\begin{vmatrix} -2 & 1 \\ 4 & 2 \end{vmatrix} + 0\begin{vmatrix} -2 & -5 \\ 4 & 4 \end{vmatrix}$$

$$= 2(-10-4) - 3(-4-4) + 0 = -28 - 3(-8)$$

$$= -28 + 16 \neq 0$$

$\Rightarrow$ vectors are linearly independent.

(2) Find the eigenvalues and eigenvectors for the matrix

$$\mathsf{A} = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}.$$

1st eigenvalue. $\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & -2 \\ 3 & -4-\lambda \end{vmatrix}$

$$= (1-\lambda)(-4-\lambda) + 6$$

$$= (\lambda^2 - \lambda + 4\lambda - 4 + 6) = \lambda^2 + 3\lambda + 2 = 0$$

$$\Rightarrow (\lambda+2)(\lambda+1) = 0$$

eigenvalues are $\lambda_1 = -2$ $\lambda_2 = -1$

Now eigenvectors.

For $\lambda_1 = -2$, do row reduction

$$\left[\begin{array}{cc|c} -1 & -2 & 0 \\ 3 & -6 & 0 \end{array}\right] \xrightarrow[(2) = 3(1) + (2)]{(1) = -1(1)} \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right].$$

$\rightarrow x_1 = -2x_2$

let $x_2 = 1 \rightarrow$ eigenvector is $\dot{\bar{x}}^1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

For $\lambda_2 = -1$, do row reduction on

$$\left[\begin{array}{cc|c} 2 & -2 & 0 \\ 3 & -3 & 0 \end{array}\right] \xrightarrow[(2) = -\frac{3}{2}(1) + (2)]{(1) = \frac{1}{2}(1)} \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\rightarrow x_1 = x_2$ let $x_2 = 1$

$\rightarrow$ eigenvector is $\bar{x}^2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$