Worksheet #20

Consider the first order system

$$x' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} x$$

(1) Find the general solution and describe the behavior as $t \to \infty$.

1- Find eigenvalues. $\begin{vmatrix} 2-\lambda & -1 \\ 3 & -2-\lambda \end{vmatrix} = (2-\lambda)(-2-\lambda) + 3 = 0$

$\to \lambda^2 - 4 + 3 = 0 \to \lambda^2 - 1 = 0$

$\lambda = \pm 1$

2- Find eigenvectors.

For $\lambda_1 = 1$ $\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 3 & -3 & 0 \end{array}\right] \to x_1 = x_2$ $\bar{w}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

For $\lambda_2 = -1$ $\left[\begin{array}{cc|c} 3 & -1 & 0 \\ 3 & -1 & 0 \end{array}\right] \to x_2 = 3x_1$ $\bar{w}_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\to$ General solution is $\bar{x}(t) = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t}$

as $t \to \infty$, $\bar{x} \to \pm\infty$ depending on the sign of c_1

(2) Draw the direction field and a few trajectories.