Worksheet #7

(1) Solve the intial value problem. Describe the behavior of the solution as $t \to \infty$.

$$y'' + 4y' + 5y = 0, \qquad y(0) = 1 \qquad y'(0) = 0$$

1- Characteristic eqn

$r^2 + 4r + 5 = 0$

2- Find roots

$r = \frac{-4 \pm \sqrt{16 - 4(5)}}{2}$

$= -2 \pm i$

3- write soln

$y(t) = e^{-2t}(c_1 \cos t + i c_2 \sin t)$

$y'(t) = e^{-2t}(-c_1 \sin t + i c_2 \cos t) + (-2) e^{-2t}(c_1 \cos t + i c_2 \sin t)$

$y(0) = c_1 = 1$

$y'(0) = i c_2 - 2c_1 = 0 \Rightarrow c_2 = \frac{2c_1}{i} = \frac{2}{i}$

(2) Solve the intial value problem. Describe the behavior of the solution as $t \to \infty$.

$$y'' + 4y' - 5y = 0, \qquad y(0) = 1 \qquad y'(0) = 0$$

1- Characteristic eqn.

$r^2 + 4r - 5 = 0$

2- Find roots

$(r + 5)(r - 1) = 0$

$\Rightarrow r = 1, -5$

3- write soln

$y(t) = c_1 e^{t} + c_2 e^{-5t}$

use IC to find c_1 & c_2

$y'(t) = c_1 e^{t} - 5c_2 e^{-5t}$

$y(0) = c_1 + c_2 = 1$

$y'(0) = c_1 - 5c_2 = 0$

$\rightarrow 6c_2 = 1 \rightarrow c_2 = 1/6$

$c_1 = 1 - c_2 \Rightarrow 5/6$

$y(t) = \frac{5}{6} e^{t} - \frac{1}{6} e^{-5t}$

(3) Using Euler's formula, show that $\sin t = \frac{e^{it} - e^{-it}}{2i}$.

$\frac{e^{it} - e^{-it}}{2i} = \frac{\cos t + i \sin t - (\cos t - i \sin t)}{2 \cdot i} = \frac{2i \sin t}{2i}$

$= \sin t$

1 continued

$$y(t) = e^{-2t}(\cos t + 2\sin t)$$

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} e^{-2t}(\cos t + 2\sin t) = 0.$$