Worksheet #9

Find the general solution of the differential equation.

(1) $y'' + 2y' = 4\sin 2t$

1- Find homogeneous soln.

$r^2+2r=0 \Rightarrow r=0,-2 \Rightarrow y_1(t)=1. \quad y_2(t)=e^{-2t}.$

2- Guess for particular soln.

$Y(t) = A\cos 2t + B\sin 2t$

$Y' = -2A\sin 2t + 2B\cos 2t$

$Y'' = -4A\cos 2t - 4B\sin 2t$

$Y''+2Y' = -4A\cos 2t - 4B\sin 2t + 2(-2A\sin 2t + 2B\cos 2t) = 4\sin 2t$

$= (-4A+4B)\cos(2t) + (-4B-4A)\sin(2t)$

$\Rightarrow -4A+4B=0 \Rightarrow A=B \qquad -4B-4A=4$

$-8B=4 \Rightarrow B=-1/2$

$y(t) = C_1 + C_2e^{-2t} - \frac{1}{2}\cos 2t - \frac{1}{2}\sin 2t$

(2) $y'' + 2y' + y = 2e^{-t} + 5t$

1- Find homogeneous solution.

$y''+2y'+y=0$

$r^2+2r+1=0 \Rightarrow (r+1)^2=0 \Rightarrow y_1(t)=e^{-t} \quad y_2(t)=te^{-t}$

2- Find particular soln.

Need to solve 2 non-homogeneous problems.

$y''+2y'+y=2e^{-t}$ & $y''+2y'+y=5t.$

Guess $Y_1(t)=At^2e^{-t}$ since -1 is a root of the characteristic eqn w/ multiplicity 2.

$Y_1' = A(-t^2e^{-t}+2te^{-t})$

$= Ae^{-t}(2t-t^2)$

$Y_1'' = Ae^{-t}[(2-2t)-(2t-t^2)]$

$= Ae^{-t}[2-4t+t^2]$

Plug this into the DE

$$Y_1'' + 2Y_1' + Y_1 = Ae^{-t}\left[2-4t+t^2\right] + 2Ae^{-t}(2t-t^2) + At^2e^{-t}$$

$$= Ae^{-t}\left[2 + t(-4+4) + t^2(1-2+1)\right]$$

$$= 2Ae^{-t} = 2e^{-t} \rightarrow A = 1$$

$$\Rightarrow Y_1(t) = e^{-t}t^2$$

Now we solve $y'' + 2y' + y = 5t$

Guess $Y_2(t) = At + B.$

$$Y_2'(t) = A$$

$$Y_2''(t) = 0.$$

$$\rightarrow Y_2'' + 2Y_2' + Y_2 = 2A + At + B = 5t$$

$$\rightarrow A = 5 \qquad 2A + B = 0$$

$$\rightarrow B = -10.$$

$$Y_2(t) = 5t - 10.$$

So $y(t) = c_1y_1(t) + c_2y_2(t) + Y_1(t) + Y_2(t)$

$$\boxed{y(t) = c_1e^{-t} + c_2te^{-t} + t^2e^{-t} + 5t - 10}$$