(1) A. Find the first 3 nonzero terms of series solutions corresponding to the larger root of the indicial equation at the regular singular point $x=0$ of the differential equation:

$$x^2 y'' + xy' + (x^2 - 1)y = 0$$

B. extra credit: This is Bessel’s equation of order 1. What does it have to do with the wave equation? (No more than 2 sentences, please!)
(2) Consider the equation of a vibrating string of length 10 with fixed ends at 0 and 10:

\[ u_{xx} = u_{tt} \]

Find the first three nonzero terms of the Fourier series for \( u \), given that \( u(x, 0) = 0 \) and \( u'(x, 0) = f(x) \) where \( f(x) = 1 \) if \( 1 < x < 9 \) and \( f(x) = 0 \) otherwise.
(3) Consider the system of equations:

\[ x' = -2x + By, \]
\[ y' = x + y \]

A. For what values of \( B \) does the solution have decreasing oscillations?
B. Find the general solution to the system of equations in terms of real valued functions when \( B = 25/4 \).
(4) Find all critical points of the following nonlinear system. For each critical point, linearize the system, find the eigenvalues, and indicate whether the equilibrium is stable or unstable.

\[ x'(t) = xy - 8 \]
\[ y'(t) = (x + 2)(y - 4) \]