Eigenvalues and Eigenvectors

Lecture 22

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Definitions

Diagonalizable Linear Operators

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Eigenvalues and Eigenvectors

Theorem

A linear operator T on a finite-dimensional vector space V is diagonalizable if and only if there exists an ordered basis $\beta$for V consisting of eigenvectors of T. Furthermore, if T is diagonalizable, $\beta = \{v_1, v_2, \ldots, v_n\}$ is an ordered basis of eigenvectors of T, and $D = [T]_\beta$,then D is a diagonal matrix and D_{jj} is the eigenvalue corresponding to v_j for $1 \leq j \leq n$.

Diagonalization

Definition

To **diagonalize** a matrix or a linear operator is to find a basis of eigenvectors and the corresponding eigenvalues.

How to Determine Eigenvalues?

Theorem

Let $A \in M_{n\times n}(F)$. Then a scalar λ is an eigenvalue of A if and only if $det(A - \lambda I_n) = 0$.

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- Let $A \in M_{n\times n}(F)$. The polynomial $f(t) = \det(A - tI_n)$ is called the **characteristic polynomial** of A.
- Let T be a linear operator on an n-dimensional vector space V with ordered basis β. The **characteristic polynomial** $f(t)$ of T to be the characteristic polynomial of $A = [T]_\beta$.

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- *The characteristic polynomial of A is a polynomial of degree n with leading coefficient $(-1)^n$.*
- *A has at most n distinct eigenvalues.*

How to Determine Eigenvectors?

Theorem

Let T be a linear operator on a vector space V, and let λ be an eigenvalue of T. A vector $v \in V$ is an eigenvector of T corresponding to λ if and only if $v \neq 0$ and $v \in N(T - \lambda I)$.