Mathematics 29 Take-Home Final Examination

1. (20) Let θ be the finite function such that $\mathrm{dom}(\theta) = \{0\}$ and $\theta(0) = 0$. Prove that if $\mathcal{B} \subseteq \mathcal{C}_1$ and $\theta \in \mathcal{B}$ while $\mathcal{B} \neq \mathcal{C}_1$, then $B = \{\, x : \varphi_x \in \mathcal{B} \,\}$ is productive.

2. (20) Show that there are infinitely many disjoint sets B_n such that B_n is productive.

3. (20) Find an example of $f, g \in \mathcal{C}_1$ and $\mathcal{B} \subseteq \mathcal{C}_1$ with $f \in \mathcal{B}$, and $g \notin \mathcal{B}$ but $B = \{\, x : \varphi_x \in \mathcal{B} \,\}$ is not productive.

4. (20) Problem 13, page 133.

5. (20) Problem 5, page 139.