Mathematics 29 Take-Home Midterm Examination

1. (20) Write a program for a URM that computes $f(x) = 2x + 1$.

2. (20) Find a Turing machine that computes $f(x) = 2x + 1$.

3. (20) Find a Post system that shows $f(x) = 2x + 1$ is Post-computable.

4. (20) Suppose A and B are decidable subsets of $\mathbb{N}$. Using Church's thesis, show that the set of n belonging to A but not to B is decidable.

5. (20) The Ackermann function $\psi(x, y)$ is defined by

   $\psi(0, y) = y + 1$,

   $\psi(x + 1, 0) \simeq \psi(x, 1)$,

   $\psi(x + 1.y + 1) \simeq \psi(x, (\psi(x + 1, y))$.

   Suppose the last statement is replaced by

   $\psi(x + 1, y + 1) \simeq \psi(x, \psi(x, y))$

   and then by

   $\psi(x + 1, y + 1) \simeq \psi(x + 1, \psi(x, y))$.

   Which of these two functions is computable?