Name Solutions Date

Math 2 — Quiz 6

1. Decompose the following rational function using partial fraction decomposition

$$\frac{-6x-17}{x^2-4x+4}.$$

$$\frac{-6x-17}{x^2-4x+4} = \frac{-6x-17}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2}$$

$$\Rightarrow -6x-17 = A(x-2)+B$$

$$= Ax-2A+B$$

$$\Rightarrow \begin{array}{l} -6=A \\ -2A+B=-17 \end{array} \Rightarrow \begin{array}{l} -2(-6)+B=-17 \\ 12+B=-17 \\ B=-29 \end{array}$$

So, $\boxed{\frac{-6x-17}{x^2-4x+4} = \frac{-6}{x-2} - \frac{29}{(x-2)^2}}$

2. Determine whether the improper integral is divergent or convergent. If convergent, compute its value.

$$\int_1^\infty \frac{1}{(2x+1)^3}\,dx.$$

$$\int_1^\infty \frac{1}{(2x+1)^3}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{(2x+1)^3}\,dx$$

$u = 2x+1$

$du = 2dx$

$$= \lim_{t\to\infty} \int_3^{2t+1} \frac{1}{u^3}\cdot\frac{1}{2}\,du$$

$$= \frac{1}{2}\cdot \lim_{t\to\infty} \int_3^{2t+1} \frac{1}{u^3}\,du$$

$$= \frac{1}{2}\cdot \lim_{t\to\infty} \left(\frac{u^{-2}}{-2} \Big|_3^{2t+1} \right)$$

$$= \frac{1}{2}\cdot \lim_{t\to\infty} \left(\underbrace{\frac{-1}{2}\cdot\frac{1}{(2t+1)^2}}_{\to 0} + \underbrace{\frac{1}{2}\left(\frac{1}{3^2}\right)}_{\to \frac{1}{18}} \right)$$

$$= \frac{1}{2}\cdot\frac{1}{18}$$

$$= \boxed{\frac{1}{36}}$$