Instructions: You are encouraged to work out solutions to these problems in groups! Discuss the problems with your classmates and/or your instructor. After doing so, please write up your solutions legibly on a separate sheet (or sheets) of paper (this part should be done on your own) and write down the names of the classmates with whom you worked. Be sure to use *complete sentences*. Note: Proofs should contain *words*, not just symbols.

1. Let $\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 6 & 3 \end{bmatrix}$ and $\theta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 10 & 6 & 5 & 8 & 7 & 1 & 2 & 4 & 9 \end{bmatrix}$.
   (a) Determine the orders of each of these permutations, $|\sigma|$ and $|\theta|$. Explain.
   (b) Determine σ^{11} and θ^{11}. If you do this the hard way (computing each of the 11 powers for each permutation), show your work. If you find an easier way (which I *highly* recommend doing), give a sentence or so explaining your reasoning.

2. (a) Determine whether $\eta = (1,2,3)$, $\mu = (2,4,3,1)$ and $\nu = (1,3)(2,5,4)$ are even permutations or odd permutations. Be sure to give an explanation.
   (b) (Chapter 5, Exercise 14) In S_n, let α be an $r-$cycle, β an $s-$cycle, and γ a $t-$cycle. Complete the following statements: $\alpha\beta$ is even if and only if $r+s$ is ______; $\alpha\beta\gamma$ is even if and only if $r+s+t$ is ______.
   (c) (Chapter 5, Exercise 16) Associate an even permutation with the number $+1$ and an odd permutation with the number -1. Draw an analogy between the result of multiplying two permutations and the result of multiplying their corresponding numbers $+1$ or -1.

3. (Chapter 5, Exercise 48) Show that for $n \geq 3$, $Z(S_n) = \{\epsilon\}$ (the center of S_n is trivial).

4. Theorems 6.2 and 6.3 give us a number of properties of isomorphisms. Using these, give 2 reasons justifying each of the following statements:
   (a) $D_4 \ncong S_4$.
   (b) $D_4 \ncong Q$, where Q is the group of the quaternions.

5. (a) Describe the elements of $Aut(Z_{10})$.
   (b) (Chapter 6, Exercise 20) Suppose that $\phi : Z_{50} \to Z_{50}$ is an automorphism with $\phi(11) = 13$. Determine a formula for $\phi(x)$.

6. (a) Prove that complex conjugation gives an automorphism of $\mathbb{C}^*$ under multiplication.
   (b) (Chapter 6, Exercise 10) Let G be a group. Prove that the mapping $\alpha(g) = g^{-1}$ for all g in G is an automorphism if and only if G is Abelian.

7. (Chapter 6, Exercise 34) If a and g are elements of a group, prove that $C(a)$ is isomorphic to $C(gag^{-1})$.

8. (Chapter 5, Exercise 10) Show that a function from a finite set S to itself is one-to-one if and only if it is onto. Is this true when S is infinite? Suppose G is a finite group and $\varphi : G \to G$ is a group homomorphism. How could we use what we just proved to simplify the proof that φ is an automorphism?

9. (Chapter 6, Exercise 6) Prove that the notion of group isomorphism is transitive. That is, if G, H and K are groups and $G \cong H$ and $H \cong K$, then $G \cong K$.