Instructions: You are encouraged to work out solutions to these problems in groups! Discuss the problems with your classmates and/or your instructor. After doing so, please write up your solutions legibly on a separate sheet (or sheets) of paper (this part should be done on your own) and write down the names of the classmates with whom you worked. Be sure to use *complete sentences*. Note: Proofs should contain *words*, not just symbols.

1. Write each of the following groups as external direct products.
   (a) Write Z_{26} as the direct product of two smaller cyclic groups.
   (b) Write Z_{180} as the direct product of smaller cyclic groups in at least two different ways. (Note: these different ways will obviously be isomorphic to one another.)
2. The dihedral group of order 8, D_4, has a cyclic subgroup of order 4 (the rotations) and several subgroups of order 2. Give one reason why $D_4 \ncong Z_4 \oplus Z_2$.
3. (Chapter 8, exercise 14) Suppose $G_1 \cong G_2$ and $H_1 \cong H_2$. Prove $G_1 \oplus H_1 \cong G_2 \oplus H_2$. State the general case (if $G_1 \cong \tilde{G}_1$, $G_n \cong \tilde{G}_n$,..., $G_n \cong \tilde{G}_n$, then what can you say about the direct products of the G_i's and of the $\tilde{G}_i$'s?)
4. (Chapter 7, exercise 14) Suppose that K is a proper subgroup of H and H is a proper subgroup of G. If $|K| = 42$ and $|G| = 420$, what are the possible orders of H? Explain your reasoning.
5. (Chapter 7, exercise 24) Suppose that G is a group with more than one element and G has no proper, nontrivial subgroups. Prove that $|G|$ is prime. (Do not assume at the outset that G is finite.)
6. (Chapter 9, exercise 14) What is the order of the element $14 + \langle 8 \rangle$ in the factor group $Z_{24}/\langle 8 \rangle$?
7. Let Q be the group of the quaternions and let H be the subgroup of Q: $H = \{1, -1\}$. The Cayley table for Q is given below.
   (a) Show that H is normal in Q. (If it is helpful, you may use facts you proved in previous homework assignments.)
   (b) Make a Cayley table for the factor group Q/H and from that, decide whether Q/H is isomorphic to Z_4 or $Z_2 \oplus Z_2$.

	1	-1	i	$-i$	j	$-j$	k	$-k$
1	1	-1	i	$-i$	j	$-j$	k	$-k$
-1	-1	1	$-i$	i	$-j$	j	$-k$	k
i	i	$-i$	-1	1	k	$-k$	$-j$	j
$-i$	$-i$	i	1	-1	$-k$	k	j	$-j$
j	j	$-j$	$-k$	k	-1	1	i	$-i$
$-j$	$-j$	j	k	$-k$	1	-1	$-i$	i
k	k	$-k$	j	$-j$	$-i$	i	-1	1
$-k$	$-k$	k	$-j$	j	i	$-i$	1	-1