

Math 31: Mattress Group Worksheet

Wednesday, September 19

You've seen the mattress group M . Its elements are the **symmetries** of the mattress. In other words, the elements of M are the different ways you can flip or rotate your mattress so that after the action is performed, the mattress has returned to its bed frame. These actions will alter the mattresses orientation in space while maintaining its shape.

Note that the mattress group is **different** from (yet related to) the set of all **orientations** of a mattress within its bedframe. While there is no natural binary operation on the set of mattress orientations, there is a natural binary operation of the set of symmetries. Given two actions x and y , we obtain another action $x * y$ defined as follows: First perform action x , then perform action y .

We saw that as a set, $M = \{e, f, g, r\}$ where these elements are defined as follows.

e : do nothing!



f : flip the mattress along its long axes



g : flip the mattress along its short axes



r : rotate the mattress 180 degrees



(there is indeed a backside to this worksheet)

Investigate the algebraic structure of the mattress group (with your neighbors) by answering the following questions.

1.) Make a multiplication table for the mattress group.

2.) Is M abelian? Why or why not?

3.) For each element $x \in M$, find the cyclic subgroup generated by x . For each $x \in M$, what is the order of $\langle x \rangle$?

4.) Is the mattress group cyclic? Why or why not? Make a real-world interpretation of your answer.

5.) List all the subgroups of M . Perhaps you've already found them all....