Math 31: Rings

Wednesday October 24 2012

Investigate the following questions with your friendly neighbors.

1. Let R be a ring and $a, b, c \in R$ and c is nonzero. Suppose

$$ac = bc.$$

We'd like to "cancel the c" to conclude $a = b$. This isn't always possible. For example, in $\mathbb{Z}_6$, we have $2 \cdot 3 = 4 \cdot 3$, but $2 \neq 4$. However, under certain conditions we can cancel the c.

(i) If $c \in R^\times$, prove that $a = b$.

(ii) If R is an integral domain, prove that $a = b$.

2. Consider the ring

$$C(\mathbb{R}) = \{\text{continuous functions } f : \mathbb{R} \to \mathbb{R}\},$$

where we define $+$ and $\cdot$ on this ring by

$$(f + g)(x) = f(x) + g(x) \qquad \text{and} \qquad (f \cdot g)(x) = f(x)g(x).$$

(i) Show that $C(\mathbb{R})$ is not an integral domain. Do this by constructing zero divisors.

(ii) Find the unit group $C(\mathbb{R})^\times$.

(iii) A ring element x is called **idempotent** if $x^2 = x$. Find the idempotent elements of $C(\mathbb{R})$.