6. The Borda count is described in FAPP as if it counted 1 for the lowest ranking candidate for each voter, 2 for the next lowest, etc. As it was described in class it was 0 for the lowest, 1 for the next lowest, etc. Will these always produce the same outcome or will the outcomes sometimes be different? Defend your answer.

7. Consider the system

$$\frac{dx}{dt} = (1-x)x - cxy \quad \text{and} \quad \frac{dy}{dt} = (1-y)y - cxy$$

modeling competitive hunters. It's identical to the systems considered in class and in problem 5 except for an arbitrary constant c in the terms containing xy.

   (a) What are the equilibrium points when $c = 1$?

   (b) With $c = 1$ still, what is the determinant of the matrices arising in the linearizations of the system about the equilibrium points different from $(0, 0)$?

   (c) How do small changes in c away from 1 affect the nature of the equilibrium points of the system? Is the model reliable when $c = 1$?

8.

   (a) Construct a tournament in which no team wins all its games, and with one team having a score higher than that of all the other teams. That is, construct a tournament with no transmitter having a unique vertex with maximum score.

   (b) What is the fewest number of vertices a tournament satisfying the conditions in part (a) could have? Why? (Score sequences are helpful here.)

9. Prove that a tournament has at most one transmitter.

10. Prove that a tournament is transitive if and only if it has no cycles.

11. Show that every tournament with more than three vertices has at least one transitive triple.

12. Prove that if a tournament has n vertices $v_1, \ldots, v_n$, then

$$\sum_{i=1}^{n} (\text{od}(v_i))^2 = \sum_{i=1}^{n} (\text{id}(v_i))^2$$

where $\text{od}(v)$ = outdegree of $v = s(v)$, and $\text{id}(v)$ = indegree of v.