## Definition

A graph $G$ consists of a set of vertices $V$ and a set of edges $E$, where an edge is an unordered pair of vertices.

## Examples

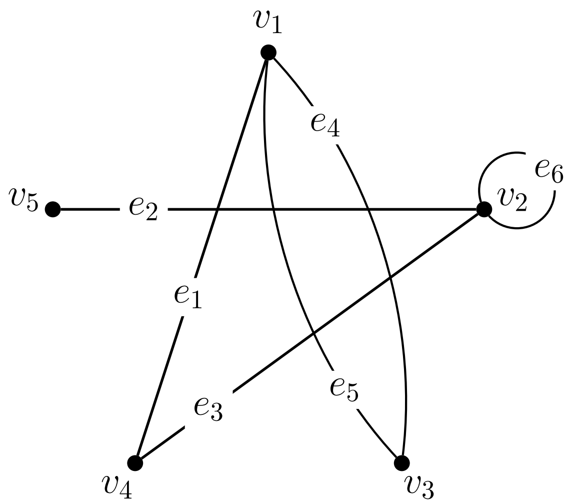
1. Computer Networks

$$V = \{\text{computers}\}$$
$$E = \{\{A, B\} \mid \text{ computers } A \text{ and } B \text{ are networked}\}$$

2. Social Networks

$$V = \{\text{Alice, Bob, Chris, Daniel, ...}\}$$
$$E = \{\{A, B\} \mid \ A \text{ and } B \text{ know each other}\}$$

3. $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.



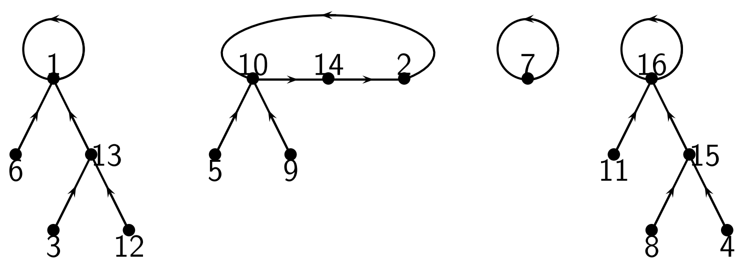
## Directed Graphs

A directed graph or digraph $G$ consists of a set of vertices $V$ and a set of directed edges $E$, where an edge is an *ordered* pair of vertices.

## Examples

1. Functional Digraphs. Let $f$ be a function from $X$ to $Y$.

$$
\begin{aligned}
V &= X \cup Y \\
E &= \{(x, y) \in X \times Y \mid f(x) = y\}
\end{aligned}
$$

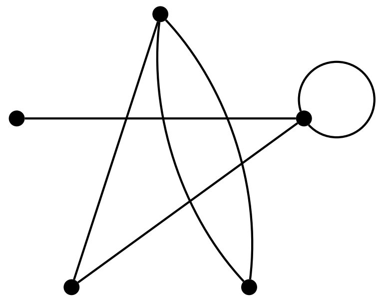


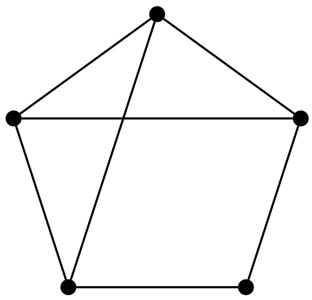

2. Game Theory

$$
\begin{aligned}
V &= \{\text{possible game states}\} \\
E &= \{(A, B) \mid \text{ state } B \text{ can be achieved from state } A\}
\end{aligned}
$$

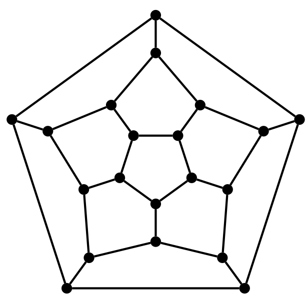
## Problems

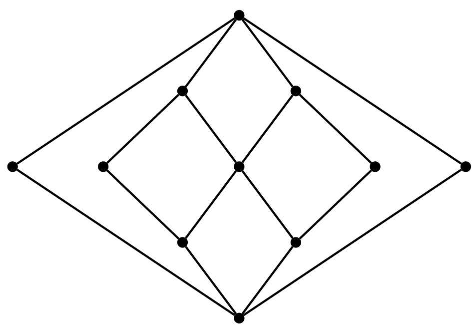
- **Eulerian Path** - Can we draw a given graph without lifting our pencil from the paper and without repeating edges?



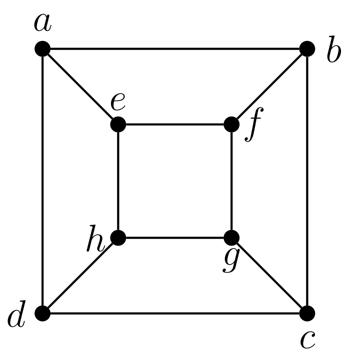


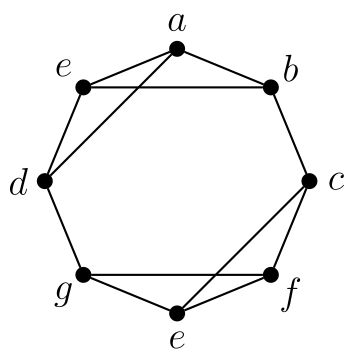
- **Hamiltonian Path** - Can we visit each vertex once and only once using only the edges of a graph?



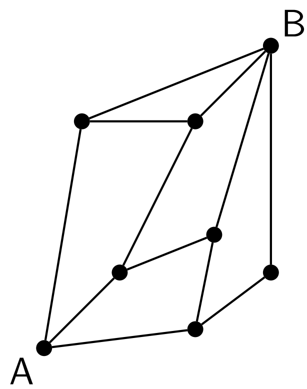


- **Isomorphism Problem** - When are two graphs the "same"?

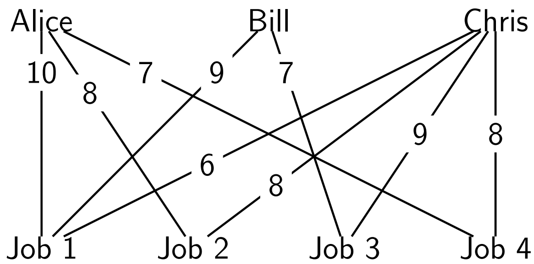


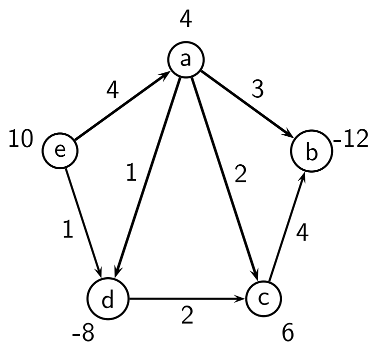

- **Shortest Route** - Find the shortest path between two vertices



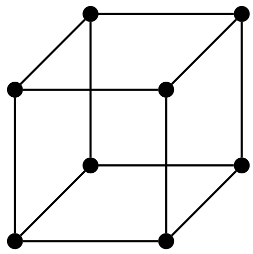
- **Assignment Problem** - How do you assign tasks to individuals so that each person can do the task assigned to them? How can you minimize time required to complete all tasks?

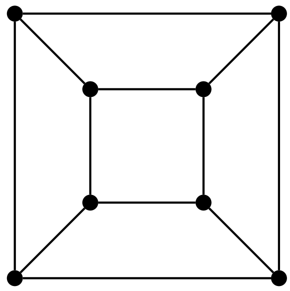



- **Network Flow Problems** - Determine the least expensive way to transport product from surplus sites to demand sites.

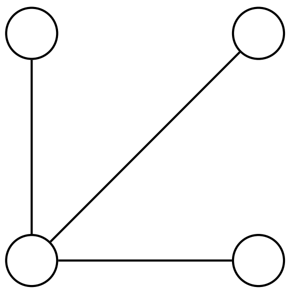


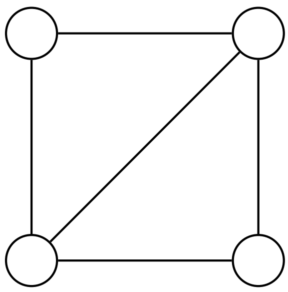
- **Planar Graphs** - Can a given graph be drawn without any pair of edges crossing? Applications to chip design.



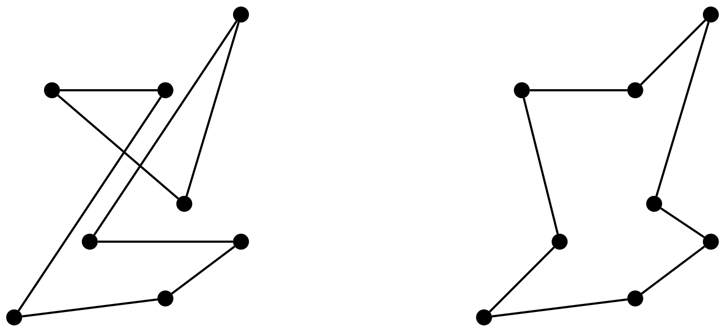


- **Chromatic Numbers** - What is the minimum number of distinct colors needed to label the vertices of a graph so that no two adjacent vertices are colored the same? Four Color Theorem.





- **Traveling Salesman Problem** - In what order should you visit cities $c_1, c_2, \ldots, c_n$ so that you minimize the distance traveled?



- **Ramsey Numbers** - What is the minimum number of people required to guarantee that there are $m$ people who all know each other or $n$ none of whom know each other

$$\begin{aligned} R(m,1) &= \\ R(2,2) &= \\ R(2,n) &= \\ R(3,3) &= \\ R(4,4) &= \\ R(5,5) &= \end{aligned}$$