Homework for Wednesday, October 4

1. Determine whether

$$(((P \to Q) \to P) \to P)$$

is a tautology.

2. Show that neither of these two wff's tautologically implies the other

$$(A \leftrightarrow (B \leftrightarrow C))$$

$$((A \wedge (B \wedge C)) \vee ((\neg A) \wedge ((\neg B) \wedge (\neg C))))$$

3. Show that $\Sigma \cup \{\alpha\} \models \beta$ iff $\Sigma \models (\alpha \to \beta)$.

4. Consider a sequence α_1, α_2, α_3 ... of wff's. For any wff φ we form a new wff φ^* by replacing each occurrence of any sentence symbol A_i in φ by the wff α_i. (So, for example, if $\varphi = (A_1 \wedge A_3)$ then $\varphi^* = (\alpha_1 \wedge \alpha_3)$.)

   (a) For any truth assignment v, we can define a new truth assignment u by setting

   $$u(A_i) = \overline{v}(\alpha_i).$$

   Show that for every wff φ,

   $$\overline{u}(\varphi) = \overline{v}(\varphi^*).$$

   (b) Show that if φ is a tautology, so is φ^*.

5. Prove the following lemma that we used in the proof of the Compactness Theorem: If Γ is a finitely satisfiable set of wff's, and α is any wff, then at least one of $\Gamma \cup \{\alpha\}$ and $\Gamma \cup \{(\neg\alpha)\}$ is finitely satisfiable.

6. In proving the Compactness Theorem, we needed to show: Suppose Δ is a finitely satisfiable set of wff's such that for every wff α, either $\alpha \in \Delta$ or $(\neg\alpha) \in \Delta$. Define a truth assignment by setting

$$v(A_i) = \begin{cases} T & \text{if } A_i \in \Delta \\ F & \text{if } A_i \notin \Delta. \end{cases}$$

Then for every wff α,

$$\overline{v}(\alpha) = \begin{cases} T & \text{if } \alpha \in \Delta \\ F & \text{if } \alpha \notin \Delta. \end{cases}$$

Prove this by induction on α.