

Homework for Wednesday, October 18

1. Show that no one of the following sentences is logically implied by the other two. (This is done by giving a structure in which the sentence in question is false, while the other two are true.)

(a) $\forall x \forall y \forall z (Pxy \rightarrow (Pyz \rightarrow Pxz))$

(b) $\forall x \forall y (Pxy \rightarrow (Pyx \rightarrow x \approx y))$

(c) $(\forall x \exists y Pxy) \rightarrow (\exists x \forall y Pxy)$

2. Show that $\{\forall x(\alpha \rightarrow \beta), \forall x\alpha\} \models \forall x\beta$. (To do this, show that any structure and variable assignment satisfying $\forall x(\alpha \rightarrow \beta)$ and $\forall x\alpha$ must also satisfy $\forall x\beta$.)
3. Show that if x does not occur free in α , then $\alpha \models \forall x\alpha$. (Hint: You may want to use the theorem we proved in class, that whether α is satisfied in a given structure with a given variable assignment s depends only on $s(v)$ for those variables v that occur free in α .)
4. Give an example of a case in which x *does* occur free in α , and $\alpha \not\models \forall x\alpha$. Show that $\alpha \not\models \forall x\alpha$ by giving an appropriate structure and variable assignment.