Final exam due 12:00 noon Friday, December 10 under the door of 105 Choate House.

1. A theory T is not finitely completable if there is no sentence σ such that $T \cup \{\sigma\}$ is complete. Show that if T is not finitely completable there is no EC U such that $T \subseteq U$ and U is complete.

2. An abelian group is a structure $(A, +)$ satisfying the following axioms:

$$\forall x\, \forall y\, \forall z\, (x + (y + z) \approx (x + y) + z)$$
$$\forall x\, \forall y\, x + y \approx y + x$$
$$\exists x\, ((\forall y\, y + x \approx y) \wedge (\forall y\, \exists z\, y + z \approx x))$$

   An abelian group is a torsion group if for every x it is true that $x \approx 0$ or $x + x \approx 0$ or $x + x + x \approx 0$ or ... similarly for every finite sum. Show that that the torsion groups are not an EC_Δ.

3. In the structure $(\mathbb{N}, +)$

   (a) define $<$.

   (b) define 0.

   (c) define 1.

4. In the structure ${}^*\mathfrak{A} = (H, \ldots)$ (where $\ldots$ has interpretations for all constants, relations and functions defined in $\mathbb{R}$ and H is the set of hyperreals) let us say that for $a, b \in H$ aEb holds if $a - b$ is finite (meaning not infinite). Show that E is an equivalence relation. For two different equivalence classes $[a]$ and $[b]$ put $[a] < [b]$ if $a < b$. Show that there is no first or last class and that the set of classes is densely ordered.

5. Using the notions of infinitesimal and standard part defined in class, and without using epsilons or deltas, explain what it means to say f is continuous