

This is a take-home midterm examination for Math 39.

You may use your class notes and the book, but you may not consult with others.

Your exam paper is due at the beginning of class Wednesday, November 8.

1. Show that the set $\{\neg, \leftrightarrow\}$ of connectives is not complete.
2. Find a sentence φ using only the sentence symbols S_1 , S_2 and S_3 which is true in every case but one. (That is in 7 out of the 8 cases the value of φ is true.)
3. Let φ be the sentence $\forall x \forall y (x \approx y)$. Let \mathcal{L} be the logic with equality and the binary relation symbol P . Show that for every sentence ψ of \mathcal{L} , either $\varphi \models \psi$ or $\varphi \models \neg\psi$. Explain your answer.
4. Consider the structures $(\mathbb{Z}, +, \cdot, 0, 1)$, $(\mathbb{Q}, +, \cdot, 0, 1)$, $(\mathbb{R}, +, \cdot, 0, 1)$, and $(\mathbb{C}, +, \cdot, 0, 1)$, where \mathbb{Z} is the integers, \mathbb{Q} is the rational numbers, \mathbb{R} is the real numbers, and \mathbb{C} is the complex numbers. For each structure find a sentence φ true in that structure and false in the others. Explain why this works. Can you do the same for these structures without the symbols 0 and 1? Once again, explain. What if there is no symbol for multiplication?