Exponential and Logarithm Functions

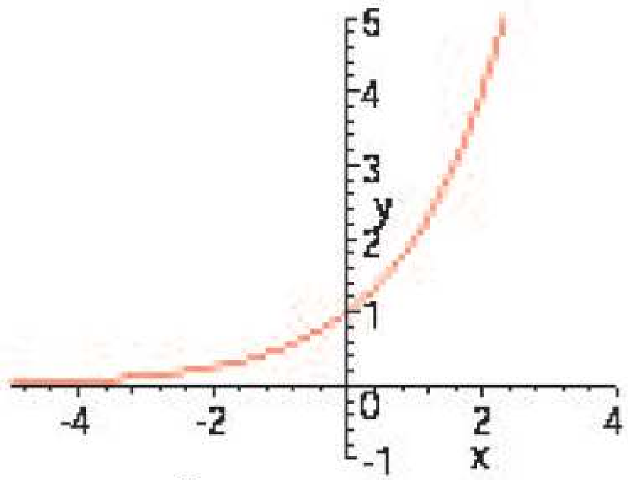
Recall:

$$
\begin{aligned}
a^n &= \underbrace{a \cdot a \cdot \ldots \cdot a}, \\
a^r &= \frac{a^m}{a^n} \text{ if } r = \frac{m}{n}.
\end{aligned}
$$

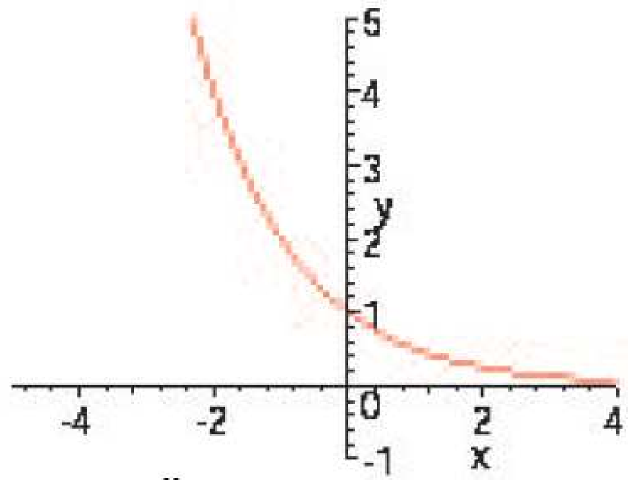
Laws of Exponents

$$
\begin{aligned}
a^0 &= 1 \qquad a^{x+y} = a^x a^y, \\
a^{-x} &= \frac{1}{a^x} \quad a^{x-y} = \frac{a^x}{a^y}, \\
(a^x)^y &= a^{xy} \quad (ab)^x = a^x b^x.
\end{aligned}
$$

Definition 1. *Let a be a positive real number. Then* $P(x) = Ba^x$ *is called a general exponential function.*



a^x for a > 1



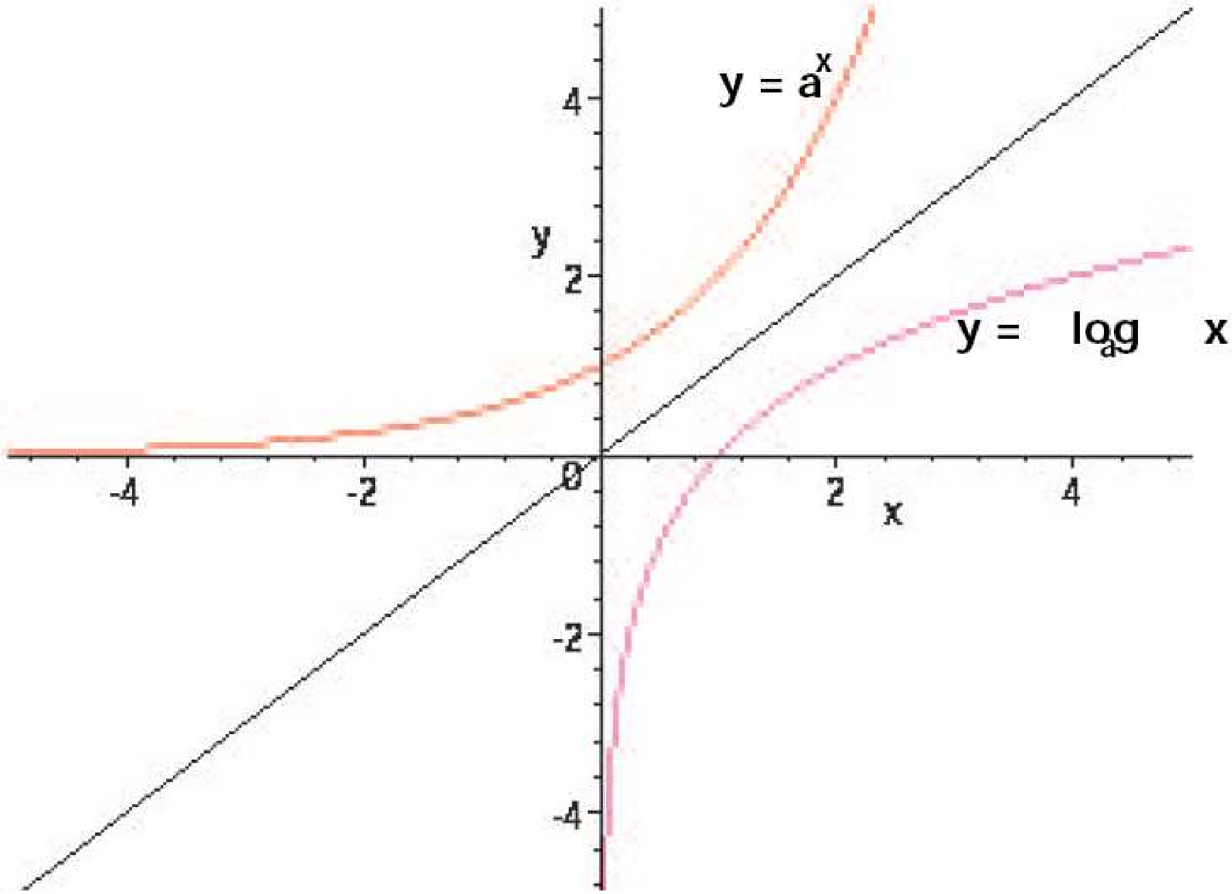
a^x for 0 < a < 1

The general logarithm function

- The inverse of the general exponential function a^x, written as $\log_a x$, is called the *general logarithm function*. It is defined by the relations:

$$y = a^x \Leftrightarrow x = \log_a y.$$

Graph of $\log x$

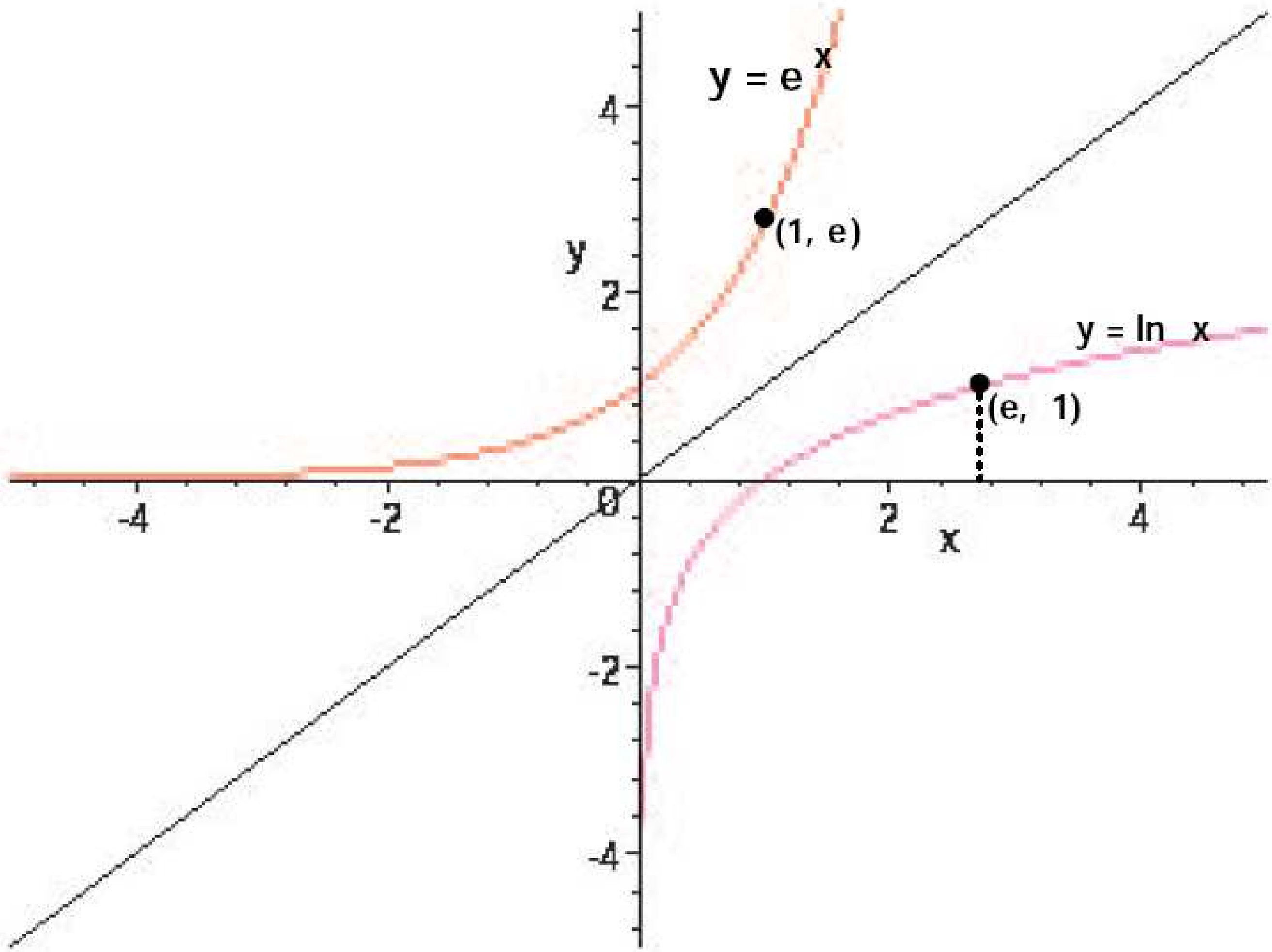


Laws of logarithms

$$
\begin{aligned}
\log_a 1 &= 0 & \log_a xy &= \log_a x + \log_a y, \\
\log_a \frac{1}{x} &= -\log_a x & \log_a \frac{x}{y} &= \log_a x - \log_a y, \\
\log_a x^y &= y \log_a x & \log_a x &= \frac{\log_b x}{\log_b a}.
\end{aligned}
$$

The natural exponential function

Definition 2. *The* natural exponential function e^x *is that exponential function that crosses the* y*-axis with slope* 1*. Its inverse* $\log_e x$ *is called the* natural logarithm function *and is denoted more simply by* $\ln x$*.*


y = e^x
(1, e)
y
y = ln x
(e, 1)
x
4
2
0
-2
-4