Step one: similar triangles

Two similar triangles have the same set of angles, and have the properties that
\[
\frac{A}{B} = \frac{a}{b}, \quad \frac{B}{C} = \frac{b}{c}, \quad \text{and} \quad \frac{A}{C} = \frac{a}{c}.
\]

Define
\[
\cos(\theta) = \frac{b}{c} \quad \text{and} \quad \sin(\theta) = \frac{a}{c}.
\]

Then let
\[
\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{a}{b}, \quad \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{c}{a},
\]
\[
\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{c}{b}, \quad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{b}{a}.
\]
Easy angles:

isosceles right triangle: equilateral triangle cut in half:

\[ \cos(\theta) = \frac{x}{1} = x \]
\[ \sin(\theta) = \frac{y}{1} = y \]

Use this idea to extend trig functions to any \( \theta \)...
Define
\[ \cos(\theta) = x \quad \sin(\theta) = y, \]
there \( \theta \) is defined by...

Sidebar: In calculus, radians are king. Where do they come from?
Circumference of a unit circle: \( 2\pi \)
Arclength of a wedge with angle \( \theta \):
\[ \frac{\theta}{180^\circ} \times 2\pi \quad \text{(if in degrees)} \quad \text{or} \quad \frac{\theta}{2\pi} \times 2\pi = \theta \quad \text{(if in radians)} \]

Reading off of the unit circle
\[ \cos(\pi - \theta) = -\cos(\theta) \quad \sin(\pi - \theta) = \sin(\theta) \]
\[ \cos(-\theta) = \cos(\theta) \quad \sin(-\theta) = -\sin(\theta) \]
\[ \cos(2\pi n + \theta) = \cos(\theta) \quad \sin(2\pi n + \theta) = \sin(\theta) \]
\[ x^2 + y^2 = 1 \implies \cos^2(\theta) + \sin^2(\theta) = 1 \]
Plotting on the \( \theta \)-\( y \) axis

Graph of \( y = \cos(\theta) \):

\[ A = \text{Amplitude} = \frac{1}{2} \text{ length of the range } = 1 \]
\[ T = \text{Period} = \text{time to repeat} = 2\pi \]

Graph of \( y = \sin(\theta) \):

\[ A = \text{Amplitude} = \frac{1}{2} \text{ length of the range } = 1 \]
\[ T = \text{Period} = \text{time to repeat} = 2\pi \]

Trig identities to know and love:

Even/odd:

\[ \cos(-\theta) = \cos(\theta) \quad \text{(even)} \quad \sin(-\theta) = -\sin(\theta) \quad \text{(odd)} \]

Pythagorean identity:

\[ \cos^2(\theta) + \sin^2(\theta) = 1 \]

Angle addition:

\[ \cos(\theta + \phi) = \cos(\theta) \cos(\phi) - \sin(\theta) \sin(\phi) \]
\[ \sin(\theta + \phi) = \sin(\theta) \cos(\phi) + \cos(\theta) \sin(\phi) \]

(in particular \( \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) \) and \( \sin(2\theta) = 2\sin(\theta)\cos(\theta) \))
Transform the graph of \(\sin(\theta)\) into the graph of \(2\sin\left(\frac{1}{2}\theta + \frac{\pi}{6}\right) - 1\):

What is the amplitude of \(2\sin\left(\frac{1}{2}\theta + \frac{\pi}{6}\right) - 1\)?

What is the period?
Other trig functions

\[ y = \cos(\theta) \]

\[ y = \sin(\theta) \]

\[ \sec(\theta) = \frac{1}{\cos(\theta)} \]

\[ \csc(\theta) = \frac{1}{\sin(\theta)} \]

\[ \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \]

\[ \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} \]