PROJECTS FOR DYNAMICAL SYSTEMS

Below is a list of possible projects for the course on Chaotic Dynamical Systems. This list are merely suggestions, you can choose other topics of a similar type. Be sure to get any project you want to do approved by the instruction of the class. This list is based on one developed by James Meiss at the University of Colorado and expanded and organized by Héctor E. Lomelí at Instituto Tecnologico Autonomo de Mexico in Mexico City.

Applied Science

1. Dynamical Systems approach to Electric Circuits

It is possible to view the equations of an Electric Circuit as a system of Ordinary Differential Equations. Some techniques of Dynamical Systems can be applied to the study of these equations. In particular, Circuits of Power Supply and Rectifiers can be studied using Lyapunov functions and multiple-scale analysis; the goal is not always to find chaos but, but in certain cases, to show stability of the system. On other cases, circuits will have chaotic behavior. Discuss both cases. This project involves the construction of the circuit described in Strogatz.

2. Chemical Patterns

There have been interesting experiments recently on patterns arising from simple chemical reactions. Could these explain the Leopard's spots and the Zebra's stripes? See Swinney, H. L. (1993). *Spatio-temporal patterns: Observation and analysis.* A. S. Weigend and N. A. Gershenfeld. *Time series prediction: Forecasting the future and understanding the past.* Reading, MA, Addison Wesley: 557-567. Another good reference is Scott, S. K. (1994) *Oscillations, Waves and Chaos in Chemical Kinetics.* Oxford Science Publications 18.

3. Mixing

The problem of mixing has to do with many Industrial applications. The interesting thing is that mixing has been studied for a long time by people in Dynamical Systems. The idea of this project is to explain what is mixing, give the basic ideas of ergodic theory. You will have to think about some industrial applications and give a possible relation of this to Fluid Dynamics.

4. Chaotic Spacecraft Orbits

In these days of smaller funding, NASA has been forced to use more complex orbits to get spacecraft to their targets. The first such orbit was for the Jacobin-Zinner comet mission (see page 101 of your textbook). The recent launch of Genesis, continues this trend with an orbit that ultimately will crash land on the earth ⟨http://genesismission.jpl.nasa.gov/⟩. There are many techniques to design such orbits; one was given in Bollt. E. and J. D. Meiss (1995), *Targeting Chaotic Orbits to the Moon Through Recurrence,* Phys. Lett. A 204, pages 373–378.

5. Small Worlds

Recently it has been discovered that many networks have the "small world" property. For example, consider the world wide web. This is a network of connected pages. It apparently can be shown that it takes no more than 19 clicks to get from any page to any other page ⟨http://physicsweb.org/article/world/14/7/09⟩. Many more examples exits. See the book Watts, D. (1999), *Small Worlds*, Princeton Univer. Press. Your task, should you accept the challenge, would be to study small worlds that have some relation to dynamics.

Economics and Finance

6. Dynamical Systems approach to Microeconomics

Explain some possible applications of Dynamical Systems to Economics. In particular, discuss why some applications to Microeconomics seem to fail. Explain why the so called "Invisible Hand" might be an idea that has no reality. This corresponds to the belief that an equilibrium allocation (zero excess demand) can be viewed as a stable fixed point solution of a suitable Dynamical System (with the introduction of a tatonnement). A good reference is Saari, Donald. (1995) *Mathematical complexity of simple economics.* Notices Amer. Math. Soc. 42 no. 2, 222-230.

7. Models of economic growth

There are many models of economic growth, generally resulting from Dynamic Optimization, that can be written in terms of a system of differential equations. For instance, one can study the models of Solow and Ramsey. Check the following books: Shone, R. *Economic Dynamics* (1997) Cambridge University Press, Klein, M. *Mathematical Methods for Economics* (1998) Addison-Wesley or Chiang, A. *Dynamic Optimization* (1992) McGraw-Hill.

8. Economic cycles

Is there an economic cycle? Why do we have periods of prosperity and periods of recession? It is not clear that economic cycles really exist. Check the following articles: Laaksonen, Matti. *Oscillations in some nonlinear economic relationships* (1996) Chaos, Solitons and Fractals 7 No. 12: pp. 2235-2245. Szydowski, M., Krawiec, A. and Tobo, J. *Nonlinear oscillations in business cycle model with time lags* (2001) Chaos, Solitons and Fractals 12: pp.505-517

9. Chaos and Finance

Some people have proposed some interesting and controversial applications of Dynamical Systems to the Research of Capital Markets. Some of the authors claim to have new points of view on how to deal with the prediction of stock prices. At the center of the controversy is the question of the validity of the Efficient Market Hypothesis, and the existence of positive Lyapunov exponents in the historical stock data. The following reference might be useful: Peters, Edgar. (1991). *Chaos and order in the Capital Markets.* Wiley. Check also a recent article in the *New Yorker*. (April 26, 1999) about *The Prediction Company.*

10. Evolutionary Game Theory

When people don't have enough information, they tend to imitate their neighbors. What happens when you can do different things in a competitive environment? This topic has applications in biology and economics. Check for instance: Friedman, D. *Evolutionary Games in Economics,* (1991) Econometrica, 59(3):637- 666.

Pure Mathematics

11. Circle Maps

Discuss Arnold's circle map $\theta \mapsto \theta + k\ \ sin(2\pi\theta)$ Interesting phenomena include mode locking intervals or tongues, and the universality near the point k = 1 for the golden mean frequency. See Chapter 4 of Strogatz and Bak, P., T. Bohr, et al. (1985). *Mode-locking and the transition to chaos in dissipative systems.* Physica Scripta 9: 50.

12. Area preserving Maps

Area preserving maps are a good model of the dynamics that one expects to find in a conservative system. Discuss the transition to chaos for an area preserving map, such as the standard map. Investigate different aspects of such maps. Numerical experiments are expected. You will have to read Meiss, J. D. (1992). *Symplectic Maps, Variational Principles, and Transport.* Reviews of Modern Physics 64(3): 795-848.

13. Kolmogorov-Arnold-Moser (KAM) Theorem

Kolmogorov proposed in 1954 a remarkable stability result for conservative (Hamiltonian) Dynamical systems. Basically, it says that such systems when weakly perturbed away from a case do not immediately become completely chaotic. This implies the possible existence, at the same time, of regular and chaotic behavior. Explain the KAM theorem, discuss its history, and the consequences for the "ergodic hypothesis of Boltzmann. (The proof is long and difficult, requiring lots of advanced mathematics).

14. Hilbert's 16th Problem

Hilbert conjectured that the number of limit cycles for a polynomial differential equation on the plane is finite. Remarkably this is still an open conjecture, though partial results have been obtained. Lookup up some of these results. Investigate models on the computer determining the number of limit cycles numerically.

15. Unimodal Maps

Discuss what the Sharkovskii sequence (the order in which periodic orbits occur) and the kneading theory tell us about the onset of chaos for unimodal, one-dimensional maps (see Devaney's book). Investigate how this changes if some assumptions (e.g., about the derivative of the map) are not satisfied.

16. Complex Dynamics

Discuss Julia Sets and the Mandelbrot Set. Do much more than copy a program for making pictures of these sets – any high school student can do this. See e.g., Peitgen, H. O., H. Jürgens, et al. (1992) *Chaos and Fractals*, New York, Springer-Verlag.

Applied Mathematics

17. Newton's Method

Show how Newton's method, in the complex domain, can have chaotic behavior. Investigate this method, or other root finders for some example functions. Numerical experiments are expected. See Benziger, H. E., S. A. Burns, et al. (1987). *Chaotic complex dynamics and Newton's method.* Phys. Lett. A 119: 442. Another good paper is Saari, D. and Urenko,J. *Newton's Method, Circle Maps, and Chaotic Motion* (1984) American Mathematical Monthly. Vol. 91, No. 1: pp. 3-17.

18. Forecasting

Nonlinear Forecasting, as introduced by Sidorovich and Farmer, is a current hot topic. In this project you will have to explain how it works and apply it to some representative data. What are the fundamental limitations on prediction implied by chaos? Data for trials is available at the Santa Fe Institute. See e.g. Sugihara, G. and R. M. May (1990). *Nonlinear Forecasting as a Way of Distinguishing Chaos from Measurement Error in Time Series.* Nature 344: 734-740.

19. Phase Space Reconstruction

Describe the Takens theory of phase space embedding and how it can be used in experiments to show the presence or absence of chaos. For references Ott, E., T. Sauer, et al., Eds. (1994). Coping with Chaos: Analysis of Chaotic Data and the Exploration of Chaotic Systems. Wiley Series in Nonlinear Science. New York, John Wiley. and Wolf, A., J. B. Swift, et al. (1985). *Determining Lyapunov exponents from a time series.* Physica D 16: 285-317.

20. Chaotic Codes

Pecora and collaborators have proposed synchronization of chaotic circuits as a technique for coding information. How does this work? Write a program to try it out. See Pecora, L. M. and T. L. Carroll (1991). *Driving systems with chaotic signals.* Phys. Rev. A 44:2374-2383.

Chaos and Fractals

21. Iterated Function Systems

Here we have a nice application of fractals to a real world problem. Iterated Function Systems (IFS) is a technique that can be used to create fractals. It can be used to compress images. For example, it seems that all the images found in some computer programs were compressed using fractals. Read the book by M. Barnsley. *Fractals everywhere.*

22. Controlling Chaos

How can one control chaos, which by its very definition is virtually unpredictable? In 1990, Ott, Grebogi and Yorke (OGY) came up with a scheme for doing this. Many papers have been written on this in the past 10 years, see Hubler, A. W. (1989). *Adaptive Control of Chaotic Systems.* Helv. Phys. Acta 62: 343-346. Also check Ott, E., C. Grebogi, et al. (1990). *Controlling Chaos.* Physical Review Letters 64(11): 1196-1199.

23. Fractal Dimension

What are the different definitions of dimension for fractals? Compute the fractal dimension using some of these techniques for some chaotic systems. See Eckmann, J.-P. and D. Ruelle (1992). *Fundamental limitations for estimating dimensions and Lyapunov exponents in dynamical systems.* Physica D 56: 185-187.

Biology

24. Biological Modeling

Dynamical Systems have been very successful in Biology. Discuss some biological models for synchronization (fireflies), population dynamics, epidemics, etc. See the book *Mathematical Biology* by Murray for possibilities. This is a research area that is growing very fast.

25. Ecology and economics

Most economic theories don't incorporate the environment. There is a need to reconciliate economic development with nature conservation. In this project you will have to propose a model that considers both the economic needs of human beings and the fragility of the natural resources. See for example: Robert V. ONeill, James R. Kahn, and Clifford S. Russell. (1998) Economics and ecology: The need for detente in conservation ecology. Conservation Ecology [on-line] 2(1): 4. Available from the Internet. ⟨http://www.consecol.org/vol2/iss1/art4⟩. Also, you might find interesting the following paper: Gro̎ller, Edward et al. *The Geometry of Wonderland.* (1996) Chaos, Solitons and Fractals. vol 7, No. 12.*

Physics

26. Modeling of Chaotic Toys

Develop and investigate a mathematical model for a chaotic toy such as

* Double Pendulum,
* Magnetic Pendulum,
* Spring-Pendulum.

27. Poincaré's solution to the three body problem

The story of Poincaré and how he gave a solution to the Restricted Three Body Problem is the story of a contest that was fixed, so he could win it. At the time, he was one of the world's leading mathematicians, and there were people that wanted him to fail. His first submission to the contest was wrong, and after the hoax was uncovered, he got a lot of pressure to write something worth the First Prize. The result was a paper in which he basically invented the whole area of Dynamical Systems. This project involves some historical research, but fortunately, two recent papers have everything that you want to know about this interesting story (see for example, Anderson, K.G. *Poincaré's Discovery of Homoclinic Points.* Archive for the History of Exact Sciences). There is also a very good book that talks about this: Barrow-Green J. *Poincaré and the Three Body Problem.* AMS.

28. The N -body problem

After Poincaré, some people continued doing research in the N -body problem. Describe some current advances in the 3,4, and 5-body problem. In particular, describe the possibility of a finite time singularity in the 5-body problem. You should show a general understanding of the N -body problem. To start, you could read Saari, Donald G. and Xia, Zhihong. (1995) *Off to infinity in finite time.* Notices Amer. Math. Soc. 42 no. 5, 538-546. Another good reference is the following: Montgomery, R. *A new solution to the three-body problem* (2001) Notices Amer. Math. Soc. 48 no. 5, 471-481.

29. Chaos in the Solar System

Some people claim that they have found some chaotic behavior in the Solar System and in some planets. This is, of course, related to the original question about the stability of the Solar System. What are the consequences for us? Are the claims true? There are several papers that deal with these questions. See for example, Kerr, R.A. *From Mercury to Pluto, Chaos pervades the Solar System.* Science 257: p.33 (July 1992), Sussman, G. and Wisdom, J. *Chaotic Evolution of the Solar System.* Science 257. P56-62 (July 1992) or Laskar, J. *A numerical experiment on the chaotic behavior of the Solar System.* Nature 338: 237-238 (March 1988).