Plastic Bottles, Q-Factors & Volume Ratios

Does the ratio between the volume of the neck of a bottle to the volume of its body correlate with the Q-factor of its resonant frequency?

Our project sought to answer this question using two distinct approaches. Our original hypothesis posited that a higher neck to body volume ratio would directly correlate to a lower Q-factor relative to other volume ratios and Q-factors measured. Having theorized this, we chose six bottles of varying neck to body ratios and measured their experimental (observed) Helmholtz resonance frequencies and the accompanying Q-factors. We expected to see a linear relationship between increasing ratio and decreasing Q-factor.

<table>
<thead>
<tr>
<th>Body Volume</th>
<th>Neck Length</th>
<th>Neck Area</th>
<th>Neck Volume</th>
<th>N:B Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>.002 m³</td>
<td>0.035</td>
<td>0.0003801</td>
<td>0.00001368</td>
<td>0.00684</td>
</tr>
<tr>
<td>.001 m³</td>
<td>0.028</td>
<td>0.0003801</td>
<td>0.000010263</td>
<td>0.010263</td>
</tr>
<tr>
<td>.00075 m³</td>
<td>0.0175</td>
<td>0.000212</td>
<td>0.00003847</td>
<td>0.051293333</td>
</tr>
<tr>
<td>.00045 m³</td>
<td>0.026</td>
<td>0.000774</td>
<td>0.00001944</td>
<td>0.0432</td>
</tr>
</tbody>
</table>

Determine Neck:Body Volume Ratio

By measuring the neck length, neck area, neck volume, and body volume for each bottle, we could determine the ratios.
Determine Natural Frequencies

With a microphone in each bottle, and Audacity's white noise, we measured the response signal of each of these bottles.

<table>
<thead>
<tr>
<th>Volume (m³)</th>
<th>Natural Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000237</td>
<td>2 L - 778.9 Hz</td>
</tr>
<tr>
<td>0.000118</td>
<td>1 L - 142.87 Hz</td>
</tr>
<tr>
<td>0.0000003</td>
<td>750 mL - 330.75 Hz</td>
</tr>
<tr>
<td>0.00000095</td>
<td>450 mL - 263.35 Hz</td>
</tr>
<tr>
<td>0.00000095</td>
<td>237 mL - 335.45 Hz</td>
</tr>
<tr>
<td>0.00000095</td>
<td>4 oz. - 278.3 Hz</td>
</tr>
</tbody>
</table>

2 L – 778.9 Hz

1 L – 142.87 Hz

750 mL - 330.75 Hz

450 mL – 263.35 Hz

237 mL – 335.45 Hz

4 oz. – 278.3 Hz
Determine Frequency Ranges

We used these same graphs to estimate the frequency at a 3-dB change in intensity in either direction, in order to determine the frequency range. We double-checked this estimate by producing a tone at the end of this frequency range and measuring the amplitude of it in comparison to the amplitude of the resonant frequency. The amplitude should be approximately \(1/\sqrt{2}(=0.707)\) in comparison to the resonant amplitude.

\[
\begin{align*}
2 \text{ L} & \cdot f_0 \pm 15 \text{ Hz} \\
1 \text{ L} & \cdot f_0 \pm 5.41 \text{ Hz} \\
750 \text{ mL} & \cdot f_0 \pm 2.25 \text{ Hz} \\
450 \text{ mL} & \cdot f_0 \pm 7.9 \text{ Hz} \\
237 \text{ mL} & \cdot f_0 \pm 13.04 \text{ Hz} \\
4 \text{ oz.} & \cdot f_0 \pm 8.85 \text{ Hz}
\end{align*}
\]

Determine the Q-Factor

Using the data from above and the formula, \(Q = f_0/\Delta f\), we determined the Q-factor for each bottle.

\[
\begin{align*}
2 \text{ L} &= 24.340625 \\
1 \text{ L} &= 13.20425139 \\
750 \text{ mL} &= 60.13636364 \\
450 \text{ mL} &= 16.66772152 \\
237 \text{ mL} &= 12.86234663
\end{align*}
\]
4 oz = 15.72316384

**Compare the Data**

We plotted the neck to body volume ratios versus the Q factor in excel to see if we could see a pattern. The Q factors are on the y-axis; the neck to body volume ratios are on the x-axis.

![Graph showing data points]

Our efforts using different bottle shapes and sizes yielded what appeared to be uncorrelated results, summarized by the graph above. We thought there could be a few explanations for the disconnect between our hypothesis and the results, the first of which being that perhaps the Q-factor was related solely to the area of the neck to body volume ratio (i.e. the size of the opening vs. the size of the bottle). However, there was no apparent correlation. Our second inclination was that perhaps we had not controlled error properly. The main issue that we didn’t control was the size and shape of the bottles. The effective length of the neck considering the tapered width was somewhat subjective. This is because for several of the bottles it was difficult to determine where exactly we should consider the beginning of the neck. Another consideration is the shape of the bottles. After some research, we discovered that the irregular shapes of the bottles, particularly the flask, made them less-than-ideal Helmholtz resonators. Or rather, though they still made effective resonators, the observed frequency differed significantly due to
the differing geometries of the bottles.¹

**Helmholtz Frequencies**

Due to the inordinately high Q-factors we measured for the 750 mL bottle and the two-liter soda bottle, we decided to calculate the theoretical Helmholtz frequency of each bottle, and compare the calculated frequencies with the observed frequencies. It is possible, for example, that we were actually calculating the resonance of a higher mode, which would explain the high frequency we recorded for the two-liter bottle.

\[
\frac{340 \times \sqrt{\left( \frac{\text{Area of Neck}}{\text{Volume of Body} \times \text{Length of Neck}} \right)}}{2\pi} \]

For the 2 L bottle, based on this formula, we seem to have picked up on a higher mode in our attempt to calculate the natural frequency of the bottle.

For the 750 mL bottle, we posit that the Helmholtz frequency is so high because the neck length is so low, especially in comparison to the neck area. The odd shape of the neck thus affects the resonating frequency.

**Adjusting for Error**

The only way to determine whether or not our hypothesis was valid was to control for the size and shape of the body. Thus we decided to conduct a second round of experiments that controlled the body volume and shape while altering the solely the neck length. Using the two-liter bottle, we added a cardboard neck of the same radius of the original neck. We used adjusted

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the neck to three different lengths, measuring the frequency and Q-factor of each with the same method described previously. We used the same methods to find the natural frequencies, the frequency ranges and the Q-factor.

**Longest Neck Length** - 0.123 m, \( f_0 = 762.8 \text{ Hz}, \Delta f = 21.88 \text{ Hz} \)

**Middle Neck Length** - 0.082 m, \( f_0 = 760.53 \text{ Hz}, \Delta f = 14.94 \text{ Hz} \)

**Shortest Neck Length** - 0.044 m

**Graph Missing, Bottle deconstructed**

\( f_0 = 764.02 \text{ Hz} \)

\( \Delta f = 7.9 \text{ Hz} \)

The plot of the neck:body ratios (x-axis) versus the Q-factor (y-axis) shows a much stronger correlation than previously. This correlation corresponds with our hypothesis that the Q factor gets lower, as the neck:body volume
ratio decreases. Despite this seemingly conclusive information, the only part of the neck:body volume ratio that we truly altered was the neck length; therefore, the correlation cannot be attributed to neck volume without taking into account the neck shape.

The observed resonating frequencies for these bottles resembled one another very closely, considering that the change in neck length, a key element in the Helmholtz frequency formula. Again, we seem to have observed higher modes than the fundamental resonant frequency of the bottles, altering the Q-factor.

**Conclusion**

Our experiment clearly did not account for all possible error nor did it isolate satisfactorily each possible contributing factor in determining Q-factor. We can say conclusively that there is a loose inverse correlation between neck:body volume ratios and q-factors for Helmholtz resonators. To examine these other factors, it would have been useful to keep the same bad size but to alter the neck width as well as the neck length, separately, to determine to what degree each of these factors had on correlating with the Q-factor.
Bibliography: