

## Math 5: Music and Sound. Homework 6

due Wed Nov 2 ... but best if do relevant questions after each lecture

1. Sketch the first five modes of an ideal string, labelling nodes (N) and antinodes (AN).
  - (a) Imagine the string is plucked  $2/5$  of the way along. Which excitation amplitudes  $\alpha_1, \alpha_2, \dots$  will be zero? Calculate the first few other values (you can ignore the  $1/n$  factor), and once you notice it repeating, state the pattern.
  - (b) Say the fundamental is 330 Hz (the top guitar E string). Sketch the spectrum (partials and their strengths) produced by this plucked string.
  - (c) Instead you place your finger lightly (damp)  $1/4$  of the way along the string while plucking the string at some general place. Which frequencies will be present? What pitch will you perceive?
2. Assume strumming an electric guitar excites the first 6 modes of a single string equally strongly ( $\alpha_1$  through  $\alpha_6$  are equal; as you know this is not true, but it is a useful idealization), and assume the string is 'open' (no fingers). Consider two electric pickup positions: A is  $1/3$  of the way along the string, whereas B is  $1/12$  of the way along.
  - (a) Use diagrams of string mode shapes (or the mode shape formula  $u_n(x) = \sin(n\pi x/L)$ ) to estimate the strengths of the partials in the signals recorded by pickup A and pickup B.
  - (b) Sketch the two spectra and discuss resulting difference in timbre.
  - (c) What spectrum will you get using pickup A if you damp (lightly touch the open string) at  $2/3$  of the way along and strum/pluck at a general place? [Harder: think about nodes]
3. Guitar builders use a 'rule of eighteen' to choose where to place each fret relative to the last one (eg Sec. 3.15 of Loy book, or this: <http://www.cybozone.com/luthier/instruments/fretscale.html>). What is the error in cents between the semitone produced by the rule of eighteen and the usual equal-tempered semitone? To get this result, what do you have to assume about string tension when the finger presses hard on the fret?
4. In the file `tube_slaps.wav` available on the homework page I recorded the sounds of two tubes A then B being slapped (the air columns excited). Use your detective skills to deduce for each one, the length, and whether the ends are open or closed. [Ignore end corrections.]
5. [changed] Lengths of wind instruments and their keyhole positions.
  - (a) A *bass flute* (yes, this exists!) has a lowest note of C3. Compute the length this flute needs to be, remembering the flute behaves like an open-open pipe [ignore end corrections].
  - (b) How far from the (non-mouthpiece) end of this flute should a finger-hole be placed so that when it opens the pitch goes up by a whole tone?
  - (c) Compute the lowest note of a soprano (so-called 'Eb') clarinet which is 49 cm long. [Hint: a clarinet behaves like a closed-open tube]. Compare your calculation to its actual lowest note G3—what could explain the difference [Hint: think about the width].
  - (d) Where on this clarinet would an ideal place be to put the register key to cause this lowest note to jump up to the *third* pipe mode ( $n = 3$ )? Assuming the lowest note G3, what notes would jumping to the second, then third, modes produce?
6. [changed] A small empty bottle has volume  $500 \text{ cm}^3$ , and a cylindrical neck of length 5 cm and diameter 19.5 mm.

- (a) Compute the Helmholtz resonant frequency [Hint: convert everything to m, m<sup>2</sup>, m<sup>3</sup> first!]
  - (b) How much (cm<sup>3</sup>) water should be poured into the bottle so that the resonant frequency changes by a perfect fifth (use 3:2)? Does it go up or down in frequency?
  - (c) With the empty bottle of (a), a second hole with another neck is added (which is equivalent to doubling the area of the neck). By what musical interval does the resonant frequency change from (a), and does it go up or down? (This is essentially what an ocarina player does when opening a new finger hole).
  - (d) Returning to the empty bottle with one neck, if the amplitude of motion of the air in the neck is 5 mm ( $5 \times 10^{-3}$  m), find the maximum speed (m/s) and maximum acceleration (m/s<sup>2</sup>) of the air in the neck [Hint: this reviews the derivation of simple harmonic motion in the mass-spring oscillator]
7. Please try to be brief (one or two sentences) but precise:
- (a) Explain why even though a flute is longer than a violin, it cannot produce a pitch as low as that of a violin.
  - (b) Explain why even though almost all glass bottles are shorter than a flute, when blown across they produce a pitch much *lower* than the lowest note of a flute. [Note: such bottles are also shorter than *half* the length of a flute; this eliminates one easy explanation!]
8. End correction of pipes (first deduced by Lord Rayleigh in 1877 !)
- (a) Imagine you have a tube of unknown length  $L$  and unknown end correction  $e$ . With both ends open the first mode has frequency 326.9 Hz but with one end closed the first mode becomes 166.7 Hz. Tell me  $L$  and  $e$ . [Hint: write one equation for each situation, then solve simultaneously].
  - (b) How long should an open-open flute be if it is to be built from a pipe of diameter 3 cm and have lowest note (all finger holes closed) C4 ? (Recall end correction is roughly 0.4 times diameter, and let's say it applies to the mouthpiece end of the flute too).