Key results:

i) Source moving

\[ f_{\text{obs}} = \frac{f}{1 - \frac{v}{c}} \quad \text{source towards you} \]

\[ f_{\text{obs}} = \frac{f}{1 + \frac{v}{c}} \quad \text{source away from you} \]

ii) Source fixed

\[ f_{\text{obs}} = f(1 + \frac{v}{c}) \quad \text{receiver towards source} \]

\[ f_{\text{obs}} = f(1 - \frac{v}{c}) \quad \text{receiver away from source} \]

Notes: In each case the 'away from' formula is gotten by setting \( v \to -v \) in the 'towards' formula.

'Fixed' and 'moving' are relative to the air (if no wind, also the ground).

In case i) above, you get a sonic boom if \( v = c \) since \( f_{\text{obs}} = \frac{f}{0} \to \infty \) !

That's it. You don't need the two-dimensional Doppler in the book (it's wrong anyway).

You don't have to know the derivation of the above.

You might have to solve for \( v \) given \( f/f_{\text{obs}} \) and \( c \).

Eg \( f = 450 \text{ Hz} \), \( v = 34 \text{ m/s towards source} \) so \( f_{\text{obs}} = \frac{450}{1 - \frac{34}{340}} = \frac{450}{0.96} = 500 \text{ Hz}, \) you hear

but if source fixed, \( v_{\text{observer}} = 34 \text{ m/s towards} \), \( f_{\text{obs}} = f(1 + \frac{v}{c}) = 450(1.1) = 495 \text{ Hz} \), you hear.

Now to clear any confusion let's derive them (as in class)…
(essence of worksheet from class)

Pulse 1 launched at $t=0$ from $x=0$.
arrives at location $x$ at time $t=\frac{x}{c}$

Pulse 2 launched at $t=T$ from location $x$.
it is already at the observer so instantly
arrives at $t=T$

Period between arrivals $T_{obs} = T - \frac{x}{c}$

But must have $x = \sqrt{T}$ by diagram.

\[ T_{obs} = T - \frac{\sqrt{T}}{c} = T \left(1 - \frac{\sqrt{T}}{T}\right) \]

using standard result $f = \frac{1}{T}$ for periodic signals.

This works for $v > 0$ (towards)
or $v < 0$ (away).

Fixed source, moving observer
(we didn't do this in class)

Pulse 1 launched at $t = -\frac{x}{c}$ at location $0$
arrives at location $x$ at time $t = 0$
Pulse 2 launched at $t = +\frac{x}{c}$ at location $0$

instantly arrives since observer also there.

So $T = \frac{x}{v} + \frac{x}{c}$ but $T_{obs} = \frac{x}{v}$

\[ \frac{f_{obs}}{f} = \frac{T}{T_{obs}} = \frac{\frac{x}{v} + \frac{x}{c}}{\frac{x}{v}} = 1 + \frac{\sqrt{T}}{v} \]

as before, $v > 0$ (towards)
or $v < 0$ (away)