An Investigation of the Baker Bells
MATH 5
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12/05/2011

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Introduction

The Baker Bells have been a Dartmouth fixture since 1928 when they first rang from the top of Baker Library. They mark the passage of time, ringing on the hour and half-hour. At 6pm every day, the sounds of the "Alma Mater" fill the campus. The bells can also be programmed to play everything from "Happy Birthday" to the Beatles. For our project, we decided to apply what we learned in class to investigate what makes the bells sound like they do.

The Baker Bells consist of 16 bells that range an octave and a half and weigh between 200 and 5300 pounds. The pitch range is from C4 to G5, with four missing notes: C#5, D#5, G#5, and D#6. Therefore, one limitation of the bells is what notes can be played. According to the Baker Bells instruction manual, only music in the keys of C, G, D, or F can be played and even in these an accidental somewhere could ruin everything.[1]

Another limitation is tempo. After a bell is struck, it takes time for the hammer to get back to its resting position before it can strike again. The striking mechanism can be somewhat slow, especially for the larger bells. Because of this limitation, repeating the same note can be a problem.

The sixteen bells have been controlled by a computer since 1979. The "Quasi Composer" is located in the control room, about 40 feet below the bells. It is capable of composing both by

[1] Grassie, Paul. *A Baker Bells Instruction Manual*, 1980.

recording from an electronic keyboard and by programming songs into the computer. The computer is connected to relays, which use electromagnets to operate the clappers of each bell (Fig. 1).

Methodology

We collect data by recording the bell sounds using high quality sound recorders. To analyze the data, we utilize the spectrum and spectrogram windows in Praat and the spectrum window in Audacity.

We make three recordings at two different locations. The first two recordings are made from the top of Baker tower. We play the same bell (Bell #6 G4) 8 times and record the sound at eight distinct positions at the corner or middle of the four sides (Fig. 2). Then we play all of the bells according to the keyboard (from low to high) and recorded each. We do not play the next bell until the echo is completely gone. There are a total of 16 tones. The third recording takes place on the third floor of Zimmerman, a residential building in the East Wheelock cluster. A small hill sits between the recorder and the bell.

For the 8-position recording, we calculate the frequencies of most partials (the strong ones) for each position, and label them according to their relative strength (Table 1)[2], to see if we can find any radiation patterns from the modes of the bells. For the 16-note recording, we

[2] This table is stored in table_1.pdf

calculate the frequencies for most partials (the strong ones) of each bell, to see if we can find any ratio patterns (Table 2)[3].

We also compare the spectra of the Zimmerman recording with the same note close-up. We choose the same time duration to generate the spectra and adjust the dynamic range to better compare the relative strength of the partials. When comparing spectrograms, we inspect with the same window length and dynamic range. Again, in order to compare the relative strength of the partials on the same scale, we adjust the frequency range to incorporate the same set of partials for each bell. (e.g., we set all the frequency upper-bound as the frequency of the 10th partial).

To verify our hypothesis that each bell can have more than one strike note, we add pure tones on top of the original recording at different times using Audacity. We adjust the frequency of the pure tone until we hear the longest beating possible. Though in theory the beating only indicates one strong partial in the spectrum, that strong partial is often found to be the actual strike note we hear.

When we study the beating of the bells, we count beatings over a long period of time to calculate beatings per second more accurately. Then we look for two close partials in the spectrum and compare the difference between the two close partials to the result of our calculation.

[3] This table is stored in table_2.pdf

Findings

I. Missing Fundamental

The phenomenon of "missing fundamental" is present in most of the 16 bells. Take Bell #1 (C4) for example. In the spectrum there is no partial at the C4 frequency. There are partials, however, sitting at multiples of C4 (Fig. 3). The 2f, 3f, 4f, and 6f, altogether, create the auditory illusion (missing fundamental) that a pitch of frequency f is heard. Similar effects are discovered in most of the other 15 bells. Because bells do not create a periodic signal, we also see strong partials at non-integer multiples of the fundamental as expected.

II. Second Strike Notes

Using the "beating method" mentioned above, we find that for many bells in the 16-note recording, there exist two pure tones that each generate a strong beating. The first beating (1067Hz) happens when the bell is just struck. Many high partials contribute to the pitch heard. Then the high partials die out very quickly and the low partial persist to create a second beating (642Hz). It is the "long tails" of the low partials that determine the second strike note. This finding confirms *The Physics of Musical Instruments*[4], which predicts a second strike note for large bells.

[4] Fletcher, Neville H, and Thomas D. Rossing. *The Physics of Musical Instruments*. New York: Springer-Verlag, 1991.

III. Beating

Due to the imperfect casting of the bell, some pairs of modes that should have the same frequency actually have different but every close frequencies (difference <15Hz). Such pair of close frequencies generates beating. Among the beatings, we find one in Bell #4 (F♯4/G4) particularly strong. That beating frequency we count (Fig. 4) matches up with the frequency difference of the two partials (Fig. 5).

IV. Obstacle and Diffraction

Looking at the spectrograms of the Zimmerman recording and the recording of the same note close-up, we see that the different locations cause different timbres. In the close-up spectrogram, there are a lot more high harmonics (Fig. 6). The spectra further reinforce this finding (Fig. 7). These observations are consistent with our predictions based on diffraction. The longer the wavelength, the easier it is for a signal to go over an obstacle. This is why we only see the lower partials in the long-distance recording.

V. Radiation Pattern

Though we cannot find any obvious radiation pattern in our recordings, a reflection on the nature of bell modes and the method we used sheds light on this failure. According to *The Physics of Musical Instruments,* the radiation pattern, if any, should be determined by *m* in the mode numbers *(m, n)* (Fig. 8) where m is half the number of nodes on the horizontal cross section.[5] However, even in the best case, we can only detect the radiation pattern for the m=2 group of modes, because once we get to a higher m, there will be too many nodes and anti-nodes that an 8-position recording will be too inaccurate to determine any pattern. Also, for the m=2 group, a pattern can only be determined if the orientation of the bell fits exactly to the locations of the recorder. It is very likely that there is an angle between the ideal recording locations and the actual ones, totally disturbing the pattern.

The mode groups mentioned above are all inextensional modes. They are inextensional in the sense that a neutral circle in each plane normal to the bell's symmetry axis remains unstretched.[6] However, there are also extensional modes. For these modes, neither nodes nor anti-nodes exist. Moreover, some of the extensional modes are not axisymmetric (Fig. 9), largely adding to the disorder of the relative strengths of the partials.

Conclusion

Some of our results agree with what the mathematical theory predicts, while others are unexpected and add more layers to our understandings of bells. Our analysis of the bells' partials line up with what we expected. We see partials at both integer and non-integer ratios of the fundamental, but with enough integer ratios to determine the missing fundamental. Our analysis of the long-distance recording also agrees with our expectations. The high frequencies are not evident when the sound has to travel through an obstacle such as a hill and trees. The bells'

[5] Ibid
[6] Ibid., pg. 676.

second strike note and beating against itself are not expected, but give us new insights on the acoustics of bells. Finally, we are unable to detect radiation pattern from the modes of the bell and have provided possible reasons for this result. We hope that our research helps to explain the sound of Dartmouth's iconic bells and inspires future investigation in the area.

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Appendix - Figures

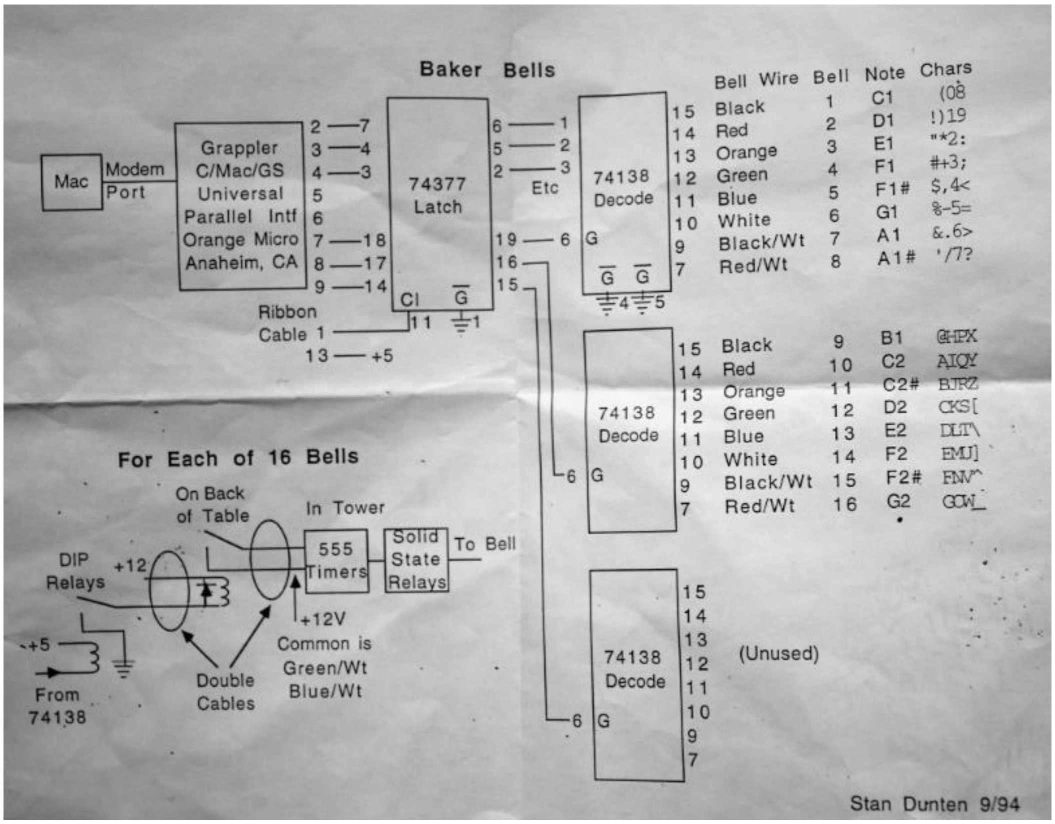


Fig. 1

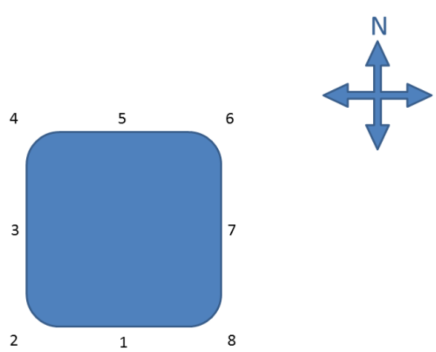


Fig. 2

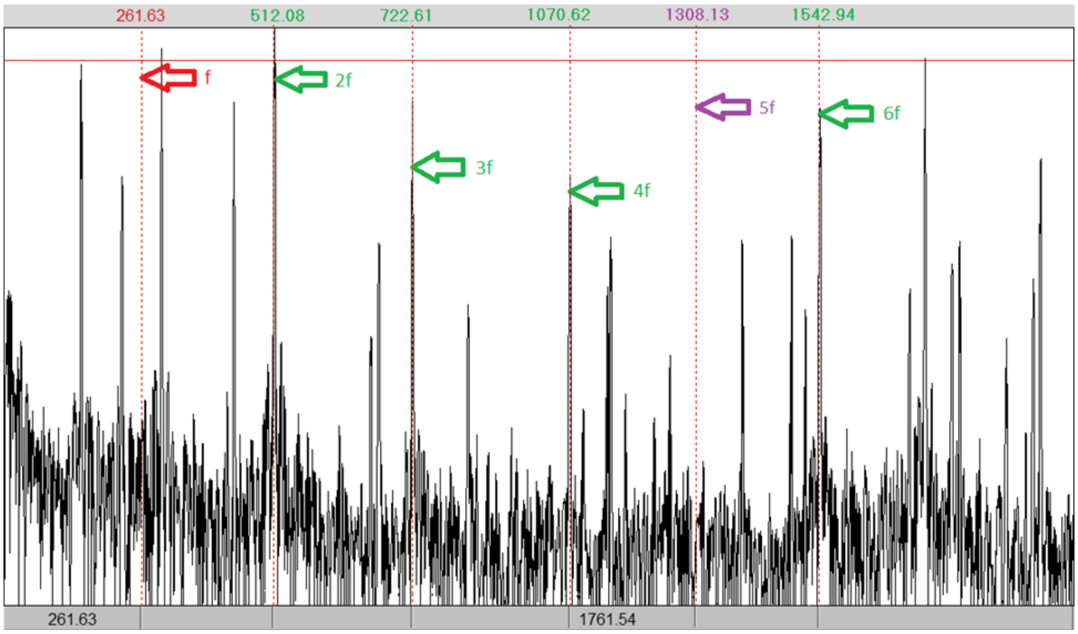
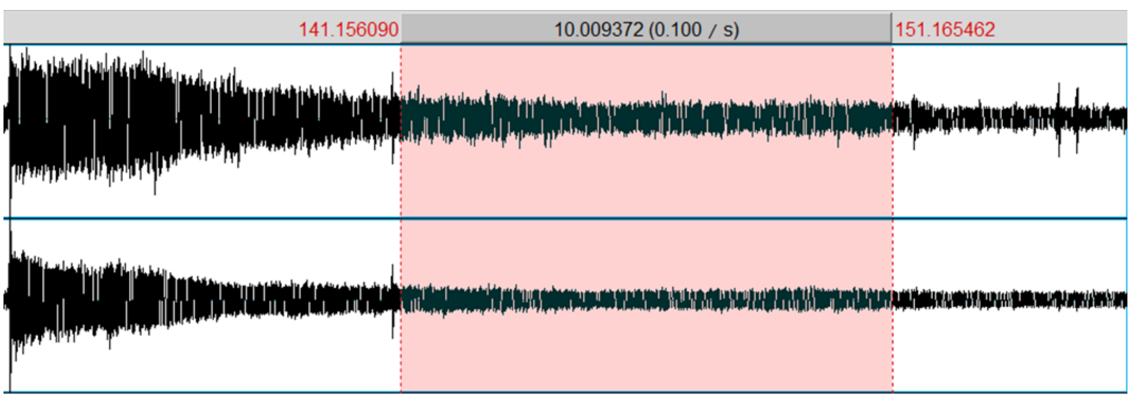


Fig. 3



$$2.77 \approx \frac{28\ beats}{10\ sec.}$$

Fig. 4

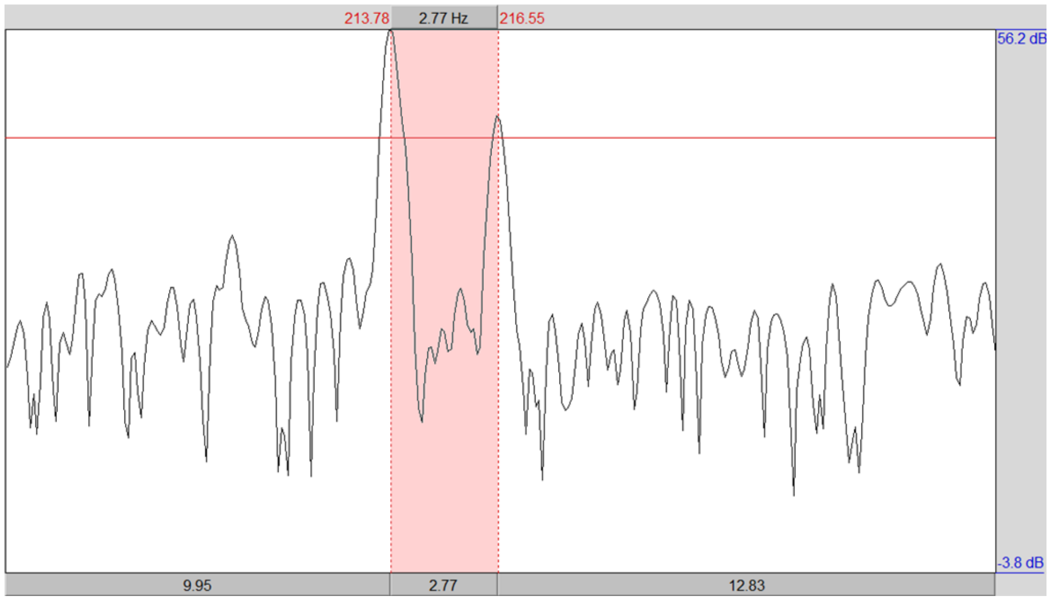

213.78
2.77 Hz
216.55
56.2 dB
-3.8 dB
9.95
2.77
12.83

Fig. 5

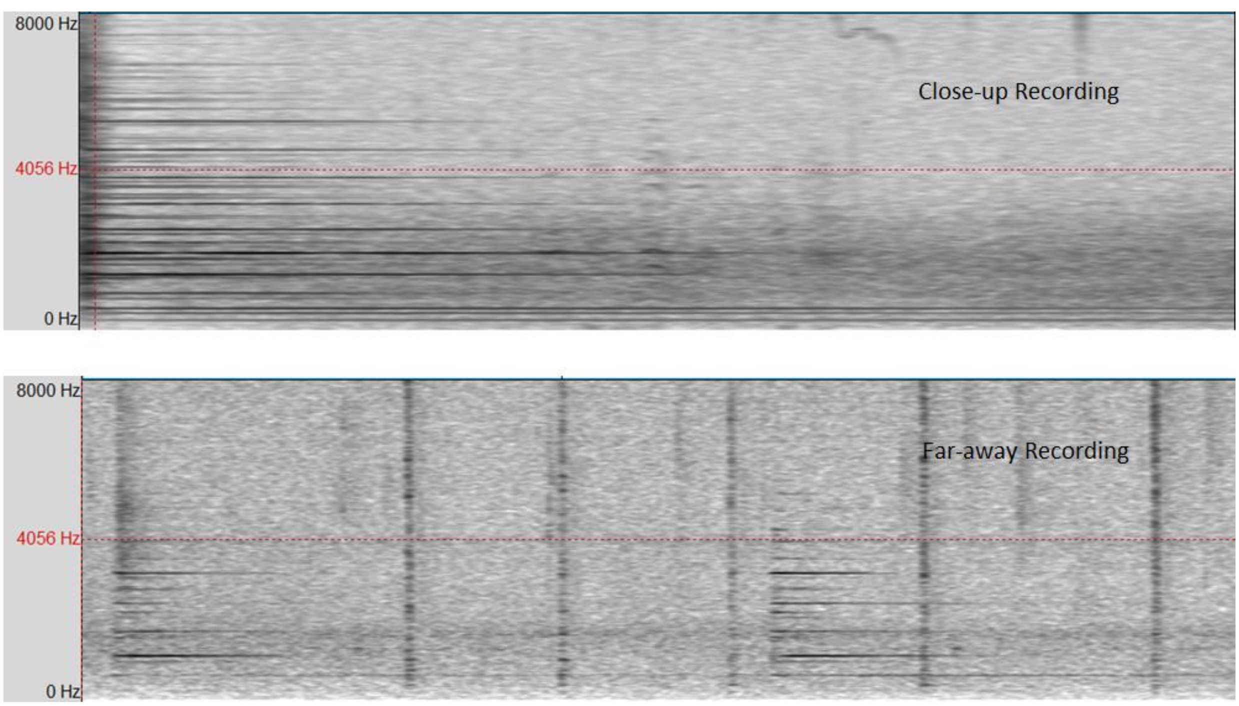

8000 Hz
Close-up Recording
4056 Hz
0 Hz
8000 Hz
Far-away Recording
4056 Hz
0 Hz

Fig. 6

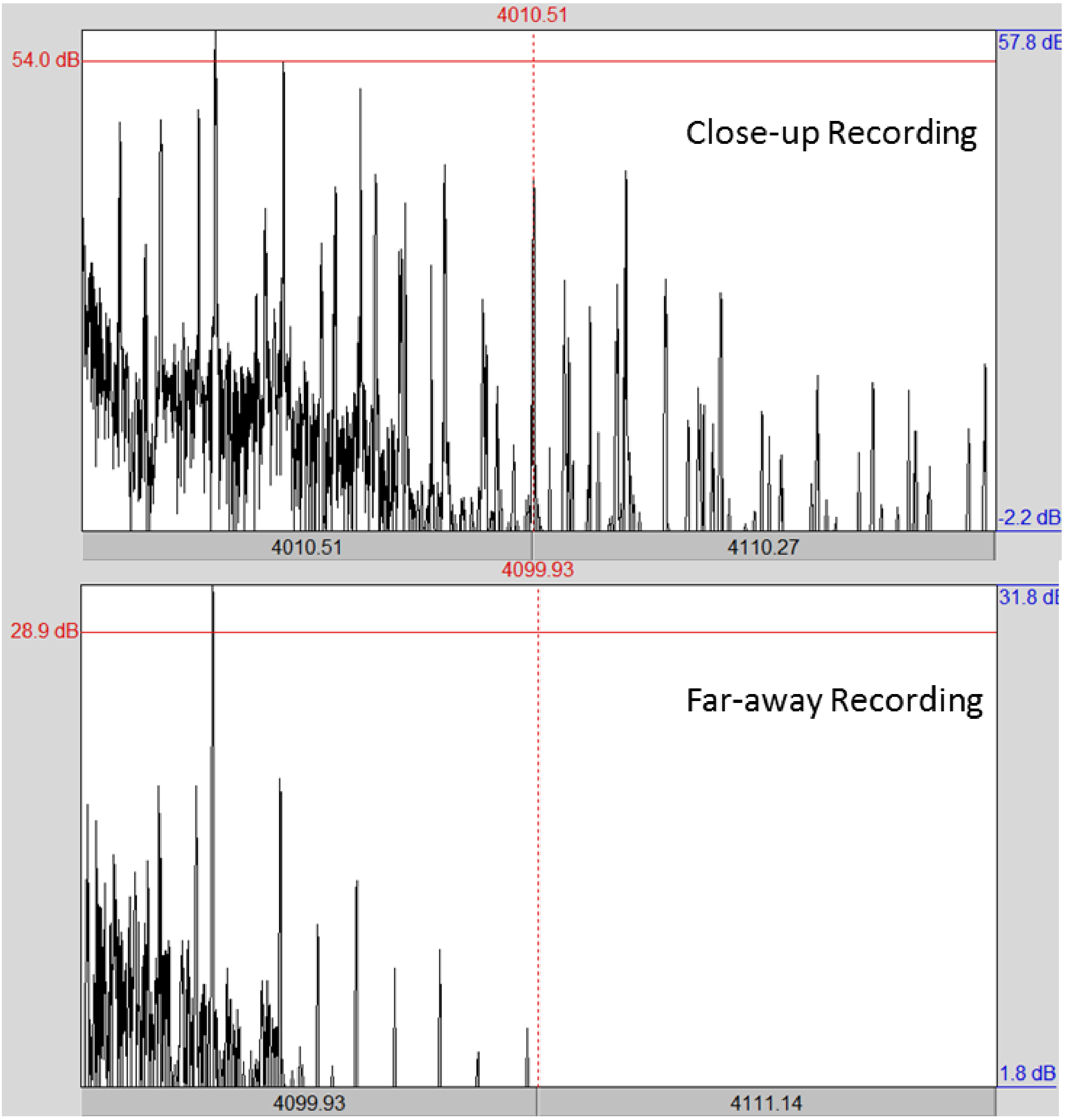

4010.51
54.0 dB
57.8 dB
Close-up Recording
-2.2 dB
4010.51
4110.27
4099.93
31.8 dB
28.9 dB
Far-away Recording
1.8 dB
4099.93
4111.14

Fig. 7

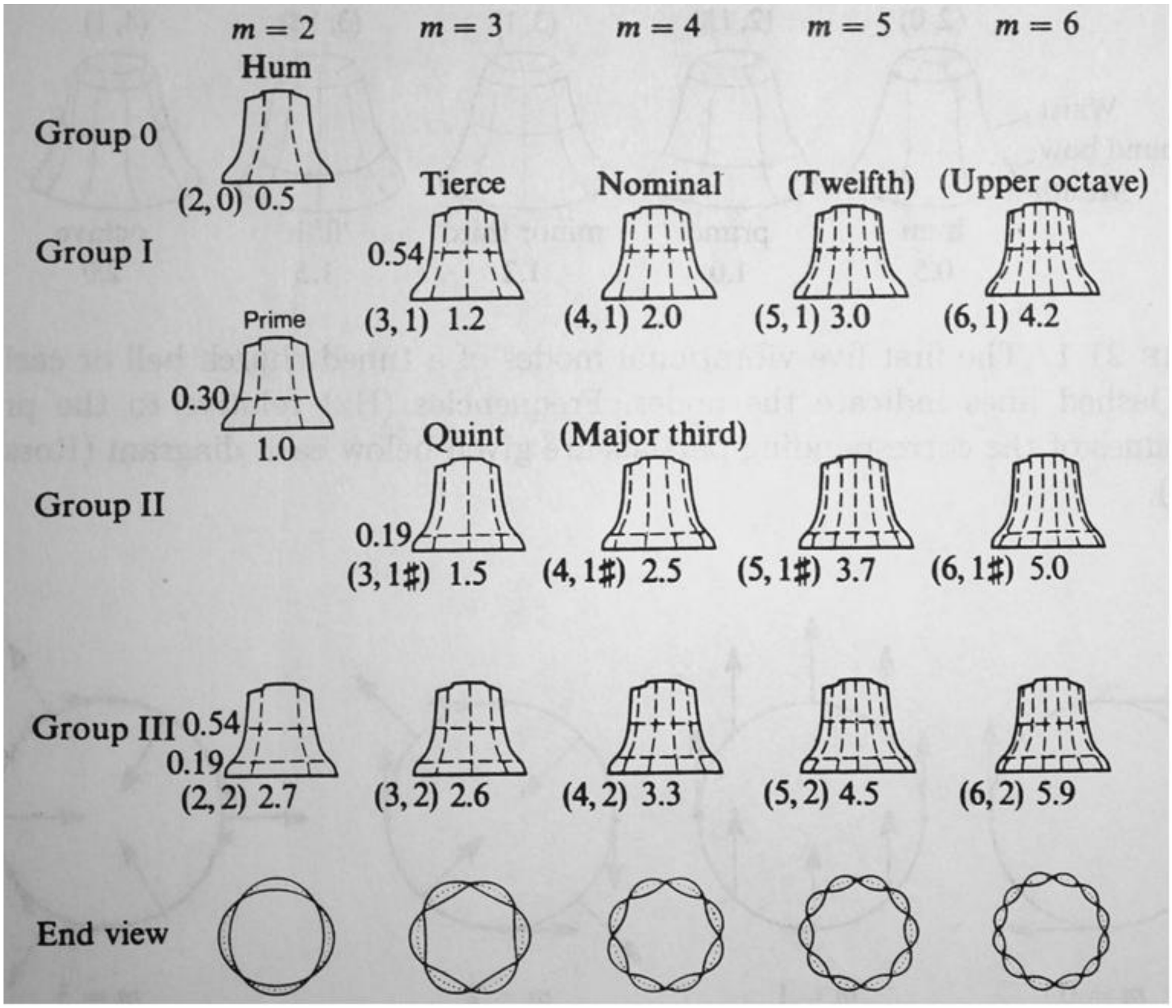

m = 2
m = 3
m = 4
m = 5
m = 6
Hum
Group 0
(2, 0) 0.5
Tierce
Nominal
(Twelfth)
(Upper octave)
Group I
0.54
(3, 1) 1.2
(4, 1) 2.0
(5, 1) 3.0
(6, 1) 4.2
Prime
0.30
1.0
Quint
(Major third)
Group II
0.19
(3, 1♯) 1.5
(4, 1♯) 2.5
(5, 1♯) 3.7
(6, 1♯) 5.0
Group III
0.54
0.19
(2, 2) 2.7
(3, 2) 2.6
(4, 2) 3.3
(5, 2) 4.5
(6, 2) 5.9
End view

Fig. 8

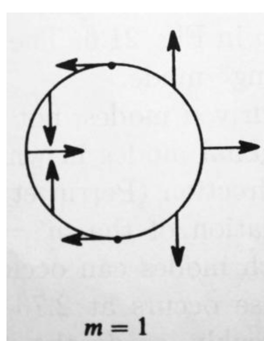

m = 1

Fig. 9