Exercises

1. Consider the set $F^n$ of all vectors $v$ with $n$ coordinates and entries in the finite field $F$ of 2 elements. We say vector $v \in F^n$ is orthogonal to vector $w \in F^n$ if the dot product $v \cdot w$ is 0.

   (a) Show that the codewords in the (8,7) parity check code are exactly the vectors in $F^8$ orthogonal to $(1,1,1,1,1,1,1,1)$.

   (b) Find 3 vectors in $F^6$ such that the codewords for the triple parity check code are exactly those vectors orthogonal to all 3 of your vectors.

   (c) Try to describe the triple repetition code in this way.

2. Show that $\mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ is a field.

3. Let $F$ be a field. Suppose $A$ and $B$ are nonzero polynomials over $F$ (that is, nonzero elements of $F[x]$). Suppose $A$ has degree $j$ and $B$ has degree $k$. Prove that the product $AB$ has degree $k + j$.

4. Let $F = \mathbb{Z}/(2)$, and let $M = x^2 + 1$ and $N = x^2 + x + 1$. Each of the systems $F[x]/(M)$ and $F[x]/(N)$ has four elements. For each system, list the four elements and write out the full $4 \times 4$ multiplication table. Exactly one of these two systems is field. Decide which one is not a field and prove that it is not.