HW #2 Math 8

1) $\displaystyle\sum_{n=0}^{\infty} \frac{2n+5}{7} \cdot (-1)^n$; $\displaystyle\sum_{n=3}^{\infty} \frac{2n-1}{7} \cdot (-1)^{n-1}$

2) $$\sum_{n=0}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=0}^{\infty} \frac{1}{3^n} + \frac{2^n}{3^n} = \sum_{n=0}^{\infty} \left(\tfrac{1}{3}\right)^n + \left(\tfrac{2}{3}\right)^n$$

$$\underset{\text{Geometric series}}{=} \frac{1}{1-\frac{1}{3}} + \frac{1}{1-\frac{2}{3}}.$$

3) $\displaystyle\sum_{n=1}^{\infty} 2^{1/n}$; Note that

$$\lim_{n\to\infty} 2^{1/n} = 2^0 = 1$$

since $1/n \to 0$ as $n \to \infty$

By Test for divergence this seq. diverges.

4) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ Note $\frac{1}{n} < \frac{1}{\sqrt{n}}$

so $\sum_{n=1}^{\infty} \frac{1}{n} \le \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

but $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges to ∞, so $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges

5) $\frac{1}{3^0} + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \dots$

$$= \sum_{n=0}^{\infty} \frac{1}{3^{2n}} = \sum_{n=0}^{\infty} \frac{1}{9^n} = \frac{1}{1-\frac{1}{9}} = \frac{9}{8}$$

geometric series

6) $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$

$$\lim_{n\to\infty} \frac{e^n}{n^2} = \lim_{x\to\infty} \frac{e^x}{x^2} = \lim_{x\to\infty} \frac{e^x}{2x} = \lim_{x\to\infty} \frac{e^x}{2}$$

diverges

L'Hopital's rule

so $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ diverges by the test for divergence

7) $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

$$\frac{2}{n^2-1} = \frac{2}{(n-1)(n+1)} = \frac{(n+1)-(n-1)}{(n-1)(n+1)} = \frac{1}{(n-1)} - \frac{1}{(n+1)}$$

$$\sum_{n=2}^{K} \frac{2}{n^2-1} = \sum_{n=2}^{K} \frac{1}{(n-1)} - \frac{1}{(n+1)}$$

$$= 1 - \cancel{\frac{1}{3}} + \frac{1}{2} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}} + \dots + \cancel{\frac{1}{K-2}} - \frac{1}{K} + \cancel{\frac{1}{K-1}} - \frac{1}{K+1}$$

$$= 1 + \frac{1}{2} - \frac{1}{K} - \frac{1}{K+1}$$

so $\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \lim_{K\to\infty} 1 + \frac{1}{2} - \frac{1}{K} - \frac{1}{K+1} = 3/2$

7) First observe that series "looks like" a geometric series:

$$\sum_{n=0}^{\infty} 7^n x^n = \sum_{n=0}^{\infty} (7x)^n$$

we know that a geometric series converges ~~for~~ when $0 \le 7x < 1$. So $0 \le x < 1/7$ must be the positive values of x that make this series converge.

8) $$\sum_{n=0}^{\infty} e^{nx} = \sum_{n=0}^{\infty} (\underbrace{e^x}_{r})^n.$$

Again this looks like a geometric series. so it will converge when $0 \le r < 1$ or $0 \le e^x < 1$. By taking ln of both sides we have $-\infty < x < 0$.

If $0 \le x$ then $1 \le e^x$ so the series div.

9) No! Let $a_n = -1$ and $b_n = 1$.

Then $$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} -1 = -\infty$$ and

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} 1 = \infty.$$

Yet, $$\sum_{n=1}^{\infty} (a_n + b_n) = \sum_{n=1}^{\infty} 0 = 0,$$ hence it converges!