Math 8
Homework Set #2

Series

1) Express the infinite sum
$$\frac{5}{7} - 1 + \frac{9}{7} - \frac{11}{7} + \cdots$$
using "sigma" notation in two different ways. First write it in the form $\sum_{n=0}^{\infty} a_n$ and also write it in the form $\sum_{n=3}^{\infty} a_n$. (Note the difference between the two is the initial value of the index n.)

Determine whether each of the following series converge or diverge. If it converges, find the sum.

2) $\displaystyle\sum_{n=0}^{\infty} \frac{1+2^n}{3^n}$

3) $\displaystyle\sum_{n=1}^{\infty} \sqrt[n]{2}$

4) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

5) $\displaystyle\frac{1}{3^0} + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \cdots$

6) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2}$

7) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1}$ (Hint: Telescoping sum)

As we have seen a Taylor series is just an "infinite polynomial" that looks like
$$f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots.$$
On the one hand, in our new language, this is just a series! On the other hand, since it contains the variable x, we may also think of it as the function, $f(x)$. For example $f(2)$ is the sum of this series obtained by replacing x with 2. (Convince yourself that $f(0) = c_0$.) Our function $f(x)$ is therefore only defined for values of x that make our series converge! Therefore, we really want to know the following: **For what values of x is $f(x)$ defined?** The next three questions deal explicitly with this.

Find the positive values of x where the following functions are defined.

7) $\displaystyle f(x) = \sum_{n=0}^{\infty} 7^n x^n$

8) $\displaystyle g(x) = \sum_{n=0}^{\infty} e^{nx}$

9) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both diverge is it necessarily the case that $\sum_{n=1}^{\infty} a_n + b_n$ also diverges?

Problems to Turn In

1) Find the positive values of x where the following function is defined.

$$h(x) = \sum_{n=0}^{\infty} (2^n + 3^n) x^n.$$

2) If $\displaystyle\sum_{n=1}^{\infty} a_n$ converges and $a_n \neq 0$ then why must the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverge?