1) $\int_0^{\pi/2} x\sin 2x\,dx$ let $u=\frac{1}{2}x$ $\quad v=-\cos 2x$

$du=\frac{1}{2}dx \quad dv=-2\sin 2x$

Then

$$\int_0^{\pi/2} x\sin 2x\,dx = \int_0^{\pi/2} u\,dv = \left[uv-\int v\,du\right]_0^{\pi/2}$$

$$= \left[-\frac{x}{2}\cos 2x - \int -\frac{1}{2}\cos 2x\,dx\right]_0^{\pi/2}$$

$$= \left[-\frac{x}{2}\cos 2x + \frac{1}{4}\sin 2x\right]_0^{\pi/2}$$

$$= -\frac{\pi}{2}\cos(\pi)+\frac{1}{4}\sin(\pi) - \left(-\frac{\pi}{2}\cos(0)+\frac{1}{4}\sin(0)\right)$$

$$= -\frac{\pi}{2}(-1)+\frac{1}{4}(0)+\frac{\pi}{2}(1)-\frac{1}{4}(0) = \underline{\pi}$$

2) $\int \underset{u}{\ln(x+1)}\,\underset{dv}{dx}$ let $u=\ln(x+1)$ $\quad V=x+1$

$du=\frac{1}{x+1}dx \quad dv=dx$

$$= \underset{v}{(x+1)}\,\underset{u}{\ln(x+1)} - \int \underset{v}{(x+1)}\,\underset{du}{\frac{1}{x+1}}\,dx$$

$$= (x+1)\ln(x+1) - \int 1\,dx = \underline{(x+1)\ln(x+1) - x + C}$$

3) $\int e^{2x}\sin x\,dx$ let $u=e^{2x}$ $\quad V=-\cos x \quad w=\sin x$

$du=2e^{2x}dx \quad dv=\sin x \quad dw=\cos x\,dx$

$$= \underset{u}{-e^{2x}}\,\underset{v}{\cos x} - \int \underset{du}{-2e^{2x}}\,\underset{v}{\cos x}\,dx$$

$$= -e^{2x}\cos x + 2\int \underset{u}{e^{2x}}\,\underset{dw}{\cos x\,dx} = -e^{2x}\cos x + 2\left(\underset{u}{e^{2x}}\,\underset{w}{\sin x} - \int 2\,\underset{w}{\sin x}\,\underset{du}{e^{2x}}\,dx\right)$$

(ctd. on next page)

3) ctd.

so

$$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x - 4\int \sin x\, e^{2x}\,dx$$

Then

$$5\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x$$

so

$$\int e^{2x}\sin x\,dx = \underline{\frac{1}{5}e^{2x}\left(2\sin x - \cos x\right) + C}$$

4) $\int x^5 \ln(x)\,dx$

$u = \ln(x)$ $\qquad v = x^6/6$

$du = \frac{1}{x}dx$ $\qquad dv = x^5 dx$

$$= \underbrace{\frac{x^6\ln(x)}{6}}_{u\cdot v} - \int \underbrace{\frac{x^6}{6}}_{v}\cdot\underbrace{\frac{1}{x}dx}_{du} = x^6\ln(x) - \int \frac{x^5}{6}dx$$

$$= x^6\ln(x) - \frac{x^6}{36} + C = \underline{x^6\left(\ln(x) - \frac{1}{36}\right) + C}$$

5) $\int_1^4 e^{\sqrt{x}}dx$

$u = 2\sqrt{x}$ $\qquad v = e^{\sqrt{x}}$

$du = \frac{1}{\sqrt{x}}dx$, $\qquad dv = \frac{1}{2}\frac{1}{\sqrt{x}}e^{\sqrt{x}}dx$

$$= \left[2\sqrt{x}\,e^{\sqrt{x}} - \int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx\right]_1^4$$

$$= \left[2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}}\right]_1^4$$

$$= 2\sqrt{4}\,e^{\sqrt{4}} - 2e^{\sqrt{4}} - \underbrace{\left(2\sqrt{1}\,e^{\sqrt{1}} - 2e^{\sqrt{1}}\right)}_{0}$$

$$= 4e^2 - 2e^2 = \underline{2e^2}$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$$

let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}dx$

$$= 2\int e^u du$$

$$= 2e^u = 2e^{\sqrt{x}}$$

6) $\int \arctan(x)\,dx$

$u = \arctan(x) \qquad v = x$

$du = \frac{1}{1+x^2}\,dx \qquad dv = dx$

$= x\arctan(x) - \int \frac{x}{1+x^2}\,dx$

$= \underline{x\arctan(x) - \frac{1}{2}\ln(1+x^2) + C}$

$\int \frac{x}{1+x^2}\,dx$ let $u = x^2$;
$du = 2x\,dx$

$= \frac{1}{2}\int \frac{1}{1+u}\,du$

$= \frac{1}{2}\ln(1+u) + C = \frac{1}{2}\ln(1+x^2) + C$

7) $\int \sin(\ln(x))\,dx$

$u = -x \qquad v = \cos(\ln(x))$

$du = -dx \qquad dv = -\frac{\sin(\ln(x))}{x}\,dx$

$= -x\cos(\ln(x)) - \int -\cos(\ln(x))\,dx$

$= -x\cos(\ln(x)) + \int \cos(\ln(x))\,dx$

Integration by parts #2:

$u = x \qquad v = \sin(\ln(x))$

$du = dx \qquad dv = \frac{\cos(\ln(x))}{x}$

$= -x\cos(\ln(x)) + x\sin(\ln(x)) - \int \sin(\ln(x))\,dx$

Then

$2\int \sin(\ln(x)) = -x\cos(\ln(x)) + x\sin(\ln(x))$

so

$\int \sin(\ln(x)) = \frac{1}{2}\underline{\left(-x\cos(\ln(x)) + x\sin(\ln(x))\right) + C}$