Math 8
Vector Valued Functions & Their Derivatives

Practice Problems

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1) Sketch the following 2-space vector function on the xy-plane:

a) $r(t) =< t, \sin t >$ b) $r(t) =< \sin t, t >$ c) $r(t) =< t^2, t^4 >$

2) Describe in words AND with a picture the curve $r(t) =< t\cos t, t\sin t, t >$.

3) Find a vector function that represents the curve of intersection of the cylinder $x^2+y^2 = 4$ and the surface $z = xy$.

4) Do Problems 21-26 on page 870 in Stewart Edition 7.

5) At what point(s) does the curve $r(t) =< 3t, 0, 2t-t^2 >$ intersect the paraboloid $z = x^2+y^2$.

6) For the following vector function, find the tangent vector AND the equation of the tangent line at the given point:

a) $r(t) =< \cos t, \sin t, t >$, at $t = 2\pi$.

b) $r(t) =< e^t, e^{-t}, -\ln t >$, at $t = 1$.

7) For the curve $r(t) =< t, 1+t^2 >$ find:

a) The points where $r(t)$ and $r'(t)$ are perpendicular.

b) The points where $r(t)$ and $r'(t)$ are pointing in the same direction.

c) The points where $r(t)$ and $r'(t)$ are pointing in the opposite direction.

8) Find the arc length of the curve $r(t) =< 3\cos^3 t, 3\sin^3 t, 6 >$, when $0 \le t \le 2\pi$.

9) Find the arc length of the curve $r(t) =< \frac{t^2}{4} - \frac{\ln t}{2}, -1, t >$, when $1 \le t \le 2$.

Problems to Turn In

1) Find a vector function $r(t)$ so that $r(0) =< 3, 0, 0 >$, $r(2\pi) =< 3, 0, 2\pi >$, and as t increases from 0 to 2π it traces out the **ellipse** $x^2 + 9y^2 = 9$ one time as it climbs up the z-axis.

2) Show that the tangent vector is always perpendicular to the radius of a circle. Hint: How do we parameterize a circle in 2-Space?

3) Find the arc length of the curve $r(t) =< t, \frac{t^3}{3} + \frac{1}{4t}, -19 >$, determined by $1 \leq t \leq 2$.