1. (14) Let

$$f(x) = \ln(1 + x^2).$$

Find the first 3 nonzero terms in the Taylor series for $f(x)$ centered at $a = 2$.

n	$f^{(n)}(x)$	$f^{(n)}(2)$	$n!$	$(x-2)^n$
0	$\ln(1+x^2)$	$\ln 5$	1	1
1	$\frac{2x}{1+x^2}$	$4/5$	1	$(x-2)$
2	$\frac{2(1+x^2)-4x^2}{(1+x^2)^2}$	$\frac{10-16}{25} = \frac{-6}{25}$	2	$(x-2)^2$

$$\ln 5 + \frac{4}{5}(x-2) - \frac{3}{25}(x-2)^2$$

2. (12) Find two unit vectors perpendicular to the plane passing through the points $P(1,1,1)$, $Q(2,0,-2)$ and $R(1,-1,1)$.

2 vectors in plane: $\overrightarrow{PQ} = \langle 1,-1,3 \rangle$

$\overrightarrow{PR} = \langle 0,-2,0 \rangle$

cross-product to get normal vector:

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -3 \\ 0 & -2 & 0 \end{vmatrix} = -6\vec{i} + 0\vec{j} - 2\vec{k} = \langle -6, 0, -2 \rangle$$

make into a unit vector:

$$|\langle -6, 0, -2 \rangle| = \sqrt{36+4} = \sqrt{40}$$

$$\left\langle \frac{-6}{\sqrt{40}}, 0, \frac{-2}{\sqrt{40}} \right\rangle = \left\langle \frac{-3}{\sqrt{10}}, 0, \frac{-1}{\sqrt{10}} \right\rangle$$

second unit vector is negation of first:

$$\left\langle \frac{3}{\sqrt{10}}, 0, \frac{1}{\sqrt{10}} \right\rangle.$$

3. (12) Let $\boldsymbol{a} = \langle 3, 4, 0\rangle$. Find the value of x such that the scalar projection of the vector $\boldsymbol{b} = \langle x, 1, 1\rangle$ onto $\boldsymbol{a}$ is 2 (i.e. $\text{comp}_{\boldsymbol{a}}\boldsymbol{b} = 2$). Also find the vector projection of $\boldsymbol{b}$ onto $\boldsymbol{a}$.

$$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{\langle 3,4,0\rangle\cdot\langle x,1,1\rangle}{|\langle 3,4,0\rangle|}$$

$$= \frac{3x+4}{\sqrt{9+16}} = \frac{1}{5}(3x+4)$$

$$\frac{1}{5}(3x+4) = 2$$

$$3x = 6$$

$$x = 2$$

$$\vec{b} = \langle 2, 1, 1\rangle$$

$$\text{proj}_{\vec{a}}\vec{b} = \left(\text{comp}_{\vec{a}}\vec{b}\right)\left(\frac{\vec{a}}{|\vec{a}|}\right) = 2\left\langle \frac{3}{5}, \frac{4}{5}, 0\right\rangle$$

$$= \left\langle \frac{6}{5}, \frac{8}{5}, 0\right\rangle$$

4. (10) Find an equation of the plane that contains the line $x = 2 + t, y = 3t, z = 1 - 2t$ and is parallel to the plane $x + 3y + 2z = -1$.

point on line $(2, 0, 1)$

normal vector to plane $\langle 1, 3, 2 \rangle$

our plane is $\parallel$ so use same normal vector

$$x + 3y + 2z = 2 + 0 + 2$$

$$x + 3y + 2z = 4$$

or $(x-2) + 3y + 2(z-1) = 0$

5. (12) Find the length of the curve with vector equation $\boldsymbol{r}(t) = \left\langle \frac{t^3}{3}, \frac{t^2}{\sqrt{2}}, t \right\rangle$ from the point $(0,0,0)$ to $\left(\frac{1}{3}, \frac{1}{\sqrt{2}}, 1\right)$.

$$\vec{r}\,'(t) = \langle t^2, \sqrt{2}\,t, 1 \rangle$$

$$|\vec{r}\,'(t)| = \sqrt{t^4 + 2t^2 + 1} = \sqrt{(t^2+1)^2} = t^2 + 1$$

$$(0,0,0) \leftrightarrow t = 0$$

$$\left(\tfrac{1}{3}, \tfrac{1}{\sqrt{2}}, 1\right) \leftrightarrow t = 1$$

$$\int_0^1 (t^2+1)\,dt = \left(\tfrac{1}{3}t^3 + t\right)\Big|_0^1 = \frac{4}{3}$$

6. (10) Find the position function $\boldsymbol{r}(t)$ of a particle that has the velocity function

$$\boldsymbol{v}(t) = \boldsymbol{i} + \sin t\,\boldsymbol{j} + t\,\boldsymbol{k}$$

with $\boldsymbol{r}(0) = \boldsymbol{j}$.

$$\vec{v}(t) = \langle 1, \sin t, t \rangle$$

$$\vec{r}(t) = \langle t, -\cos t, \tfrac{1}{2}t^2 \rangle + \vec{C}$$

$$\vec{r}(0) = \langle 0, 1, 0 \rangle = \langle 0, -1, 0 \rangle + \vec{C}$$

$$\vec{C} = \langle 0, 2, 0 \rangle$$

$$\vec{r}(t) = \langle t, 2 - \cos t, \tfrac{1}{2}t^2 \rangle$$

7. (10) Show that

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

does not exist.

$x=0 \quad \lim_{y\to 0} \frac{0}{y^2} = 0$

$y=0 \quad \lim_{x\to 0} \frac{0}{x^2} = 0$

$x=y \quad \lim_{y\to 0} \frac{y^2}{2y^2} = \frac{1}{2}$

Since two paths give distinct values for the limit, it does not exist.

8. (20) For each of the following statements, fill in the blank with the letters **T** or **F** depending on whether the statement is true or false. You do not need to show your work and no partial credit will be given on this problem.

(a) $\cos 2x = \sum_{n=0}^{\infty}(-1)^n \frac{(2x)^{2n+1}}{(2n+1)!}$

this is $\sin(2x)$

Alternatively, note they are unequal at $x=0$

ANS: F

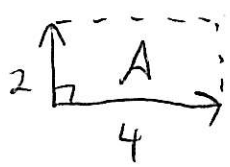
(b) Let θ be the angle between $\boldsymbol{a} = \langle 2, 2, -1\rangle$ and $\boldsymbol{b} = \langle 5, -3, 2\rangle$. Then $0 \leq \theta \leq \frac{\pi}{2}$.

$\vec{a} \cdot \vec{b} = 10 - 6 - 2 = 2$

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{pos}{pos} = pos$

ANS: T

(c) Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be two perpendicular vectors with $|\boldsymbol{a}| = 2$ and $|\boldsymbol{b}| = 4$. Then $|\boldsymbol{a} \times \boldsymbol{b}| = 8$.



$A = |\vec{a} \times \vec{b}|$

ANS: T

(d) $\lim_{t\to 0} \left\langle t, e^{-t}, \frac{\sin t}{t} \right\rangle = \langle 0,0,0 \rangle.$

neither the second nor third

component limits to zero

ANS: F

(e) The domain of the function $f(x,y) = \sqrt{4 - x^2 - y^2}$ is the set of all (x,y) such that $x \leq 2$ and $y \leq 2$.

need $x^2+y^2 \leq 4$

ANS: F