Determine which of the following series diverge, converge conditionally, and converge absolutely. Mention all tests you use. Remember that to show that a series converges conditionally, you show that $\sum a_n$ converges and also that $\sum |a_n|$ diverges (i.e., that the series does not converge absolutely).

a. $\sum_{n=1}^{\infty} \frac{n}{n+1}$

b. $\sum_{n=1}^{\infty} 10^{10} \left(\frac{2}{3}\right)^n$

c. $\sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!}$

d. $\sum_{n=1}^{\infty} \frac{(-1)^n n^4}{e^n}$

e. $\sum_{n=1}^{\infty} \frac{\sqrt{n^{10} + 2n - 1}}{\sqrt{n^9 + 2n^2}}$

f. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n + 7}}$

g. $\sum_{n=2}^{\infty} \frac{1}{n \ln n (\ln \ln n)}$

h. $\sum_{n=2}^{\infty} \frac{1}{n \ln \ln n}$

i. $\sum_{n=2}^{\infty} \frac{1}{(\ln n) \ln n}$

j. $\sum_{n=1}^{\infty} \frac{\cos n^4 + \sin n^5}{n^9}$

k. $\sum_{n=1}^{\infty} \frac{(-1)^n (n^3 + 2n + 1)}{\sqrt{n^7 - 2n + 10}}$

l. $\sum_{n=1}^{\infty} \frac{1}{n^{10} + 1}$

Find the first two nonzero terms in the Maclaurin series for $f(x) = \tan x$.

Find the Maclaurin series for $f(x) = x \arctan(3x^3)$.

Find the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{(-5)^{n+2} (x-1)^n}{n^2}$.

Evaluate the following integrals.

c. $\int \frac{1}{x^2 \sqrt{x^2 + 9}} \, dx$

d. $\int \frac{x^2}{\sqrt{1-x^2}} \, dx$

e. $\int e^{3x} \cos x \, dx$

f. $\int x \ln x \, dx$

Find the point in which the line $x = 2 - t$, $y = 1 + 3t$, $z = 4t$ intersects the plane $2x - y + z = 2$.

Determine whether the planes given by $x + 4y - 3z = 1$ and $-3x + 6y + 7z = 3$ are parallel, perpendicular, or neither. If neither, find the angle between them.

Determine whether the planes given by $3x + 6z = 1$ and $2x + 2y - z = 3$ are parallel, perpendicular, or neither. If neither, find the angle between them.
9. Find an equation of the plane which contains the $x$-axis as well as the line given by the parametric equations $x = t$, $y = 2t$, $z = 3t$.

10. Find an equation of the plane which contains the origin and the line $x = 6t + 2$, $y = 2 - 4t$, $z = 9$.

11. Find the arc length of the curve $r(t) = \cos^3 tj + \sin^3 tk$ from $t = 0$ to $t = 1$.

12. Suppose the gradient of $f(x, y, z)$ is
   \[ \nabla f = (2xyz + 2e^z, x^2z - \cos y, x^2y + 2e^z), \]
   and that $x = s^2t$, $y = t^3$, and $z = e^s$. What is $\frac{\partial f}{\partial s}$? You need not simplify your answer, but it should not contain $\partial$ symbols.

13. Consider the function $f(x, y) = x^3 + y^2 - xy$. At the point $(1, 1)$, in what direction(s) is the rate of change of $f$ equal to zero? Give your answer as one or more unit vectors.

14. Find the rate of change of the function $f(x, y) = \sqrt{24 - x^2 - y^2}$ at the point $(4, -2)$ in the direction given by $\theta = \pi/6$. In what direction does $f$ attain its maximum rate of change at the point $(4, -2)$? (You need not specify this direction by an angle.)

15. Let $f(x, y, z) = ye^{-x^2} \sin z$. Find the equation of the tangent plane to the level surface of $f$ at the point $(0, 1, \pi/3)$.

16. A ball is placed at the point $(1, 2, 3)$ on the surface $z = y^2 - x^2$. Give the direction in the $xy$-plane that the ball will start to roll.

17. Find and classify all critical points of the function $f(x, y) = 3x - x^3 - 3xy^2$.

18. Find and classify all critical points of the function $f(x, y) = x^3 + y^4$.

19. Let $f(x, y) = x \sin y$.
   a. Compute $f_x$, $f_y$, $f_{xx}$, $f_{xy}$, and $f_{yy}$.
   b. What are the critical points of $f$?
   c. Classify the critical points of $f$.
   d. Find the absolute maximum and minimum of $f$ on the region given by $-1 \leq x \leq 1$ and $-\pi/2 \leq y \leq \pi/2$.

20. Find the maximum and minimum of $f(x, y) = x^2 + 2x + y^2$ on the disk $x^2 + y^2 \leq 4$. 