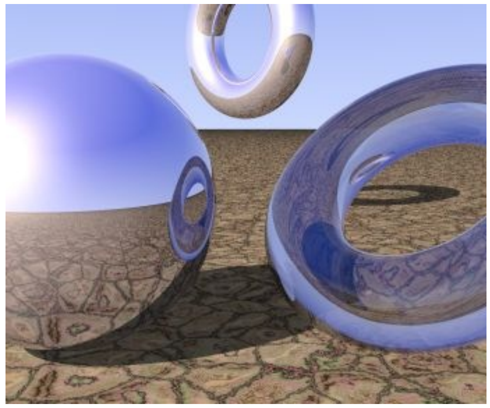
Math 112: Introduction to Riemannian Geometry
Spring 2006
T/Th 10-11:50 AM (104 Bradley)
Instructor: Craig J. Sutton



Course Description: A Riemannian manifold is a smooth manifold equipped with a Riemannian metric. With this additional structure one is able to define geometric concepts such as geodesics, connections and curvature. Geometers frequently explore the relationship between these concepts and the underlying topological and smooth structures. For example, the celebrated sphere theorem tells us that if M^n is a compact, simply-connected, smooth manifold that admits a Riemannian metric g with sectional curvature satisfying $\frac{1}{4} < \mathrm{Sec}_{(M,g)} \leq 1$, then M^n is homeomorphic to the n-sphere S^n. As a consequence, we can conclude that the universal cover of any compact manifold which admits a Riemannian metric with $\frac{1}{4} < \mathrm{Sec}_{(M,g)} \leq 1$ must be a sphere!

This term Math 112 will serve as an introduction to this classical and vibrant area of research with an aim towards developing both theoretical and computational proficiency. It should be of relevance to students with interests in geometry, topology and (mathematical) physics. The topics covered will include some of the following.

- **Riemannian Basics:** Riemannian metrics; affine & Levi-Civita connections; geodesics, the exponential map, and the geodesic flow; curvature tensor and the various curvatures; Killing fields.
- **Geometry of Isometric Immersions & Riemnnian submersions:** immersions; the second fundamental form; principal directions & curvatures; totally geodesic submanifolds; submersions; the A- and T-tensors; O'Neill's formula; principal bundles & connections.
- **Jacobi Fields & Interpretation of Curvature:** Jacobi equation and Jacobi fields; curvature and the spreading of geodesics; conjugate points and singularities of the exponential map.
- **Variations of Energy & Geodesics**
- **Spaces of Constant Curvature**
- **Manifolds of Negative Curvature**
- **Isometry Groups & Isometric Actions**
- **The Sphere Theorem**
- **The Geometry of Lie Groups and other Homogeneous Spaces**

Prerequisites: Familiarity with the basics of manifolds as covered in Math 124 or a willingness to dive in and learn as you go along.