The subject matter relevant to the graduate certification requirement have been divided into primary and secondary topics. The student is required to have a basic knowledge of the three primary topics and several of the secondary topics, the choice of which will be made together by examiners and examinee.

**PRIMARY TOPICS: Required Basic Probability**

1. Basic discrete probability, including mean and variance, common distributions, Bayes' Theorem, Borel-Cantelli Lemma.

2. Basic continuous probability, including densities, joint distributions, measure theory, convolution, characteristic functions.

3. Central Limit Theorem, and some idea of its proof; weak and strong laws of large numbers.

4. Markov chains, discrete and continuous; stationarity; hitting and mixing times; recurrence and transience.

5. Information theory and entropy.

6. Random walk, Brownian motion, law of the iterated logarithm.

7. Renewal processes, stopping times, renewal theorem.

**SECONDARY TOPICS: Additional Subjects in Probability**

1. Martingales and branching processes.

2. Statistical physics, Gibbs ensemble.

3. Randomized algorithms and their applications (e.g., primality testing).

4. The probabilistic method in combinatorics.

5. Probability amplitudes in quantum mechanics.


8. De Finetti’s Theorem and finite versions.


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1. Classical Combinatorics
   - Bogart, *Introductory Combinatorics, Second Edition* (Chapters 1, 2, 6)
   - Graham, Rothschild, and Spencer, *Ramsey Theory* (Chapter 1)
   - Liu, *Introduction to Combinatorial Mathematics*
   - Riordan, *Combinatorial Mathematics* (Chapters 2—4)
   - Stanley, *Enumerative Combinatorics, Volume 1* (Chapter 1)

2. Algebraic Techniques
   - Aigner, *Combinatorial Theory*
   - Stanley, *Enumerative Combinatorics, Volume 1* (Chapters 1—3)

3. Graph Theory
   - Bogart, *Introductory Combinatorics, Second Edition* (Chapters 4, 5)
   - Bollobas, *Graph Theory: An Introductory Course*
   - Bondy and Murty, *Graph Theory With Applications*
   - Golumbic, *Algorithmic Graph Theory and Perfect Graphs*

A. Ordered Sets
   - Birkhoff, *Lattice Theory*
   - Davey and Priestley, *Introduction to Lattices and order*
   - Stanley, *Enumerative Combinatorics, Volume 1*
   - Trotter, *Combinatorics and Partially Ordered Sets: Dimension Theory* (Chapter 3)

B. Coding Theory
   - Berlekamp, *Algebraic Coding Theory*
   - Peterson and Weldon, *Error-Correcting Codes*
   - Pless, *Introduction to the Theory of Error-Correcting Codes*
   - Sloane and MacWilliams, *The Theory of Error Correcting Codes*
   - Van Lint, *Coding Theory*

C. Combinatorial Geometry and Matroids
   - Aigner, *Combinatorial Theory*
   - Crapo and Rota, *On the Foundations of Combinatorial Theory: Combinatorial Geometries*
   - Oxley, *Matroid Theory*
   - Welsh, *Matroid Theory*

D. Matching Theory
Hall, *Combinatorial Theory*
Mirsky, *Transversal Theory: An Account of Some Aspects of Combinatorial Mathematics*
Ryser, *Combinatorial Mathematics*

E. Random Graphs and the Probabilistic Method
   Bollobás, *Random Graphs*
   Palmer, *Graphical Evolution*
   Spencer, *Ten Lectures on the Probabilistic Method*

F. Symmetric Functions
   Macdonald, *Symmetric Functions and Hall Polynomials* (Chapter 1)
   Sagan, *The Symmetric Group* (Chapters 3,4)

G. Representations of the Symmetric Group
   Sagan, *The Symmetric Group* (Chapters 1,2)